National Aeronautics and Space Administration

OPTIMAL LOW THRUST PERTURBED ORBIT TRANSFERS

Principal Investigators: Robyn Woollands (392D), John Junkins (TAMU),
Vishala Arya (TAMU), Sandeep Singh (TAMU), Ehsan Taheri (TAMU)
Program: Topic R&TD

Abstract: We present new methods for computing *fuel-* and *time-optimal* low-thrust orbit transfers. The algorithms are formulated via the indirect formalism of optimal control, which leads to a two-point boundary-value problem (TPBVP). Three main contributions are 1) to solve time-optimal low-thrust trajectories from a GTO to a quasi-frozen orbit in the restricted three-body dynamic model of the Earth-Moon system, 2) fuel-optimal trajectory design using a high-fidelity gravity model of the Earth while taking into account eclipses (no-thrust constraint), and 3) a hybrid direct/indirect scheme to combine low-thrust and gravity-assist maneuvers for fuel-optimal, interplanetary trajectory design. Hyperbolic tangent smoothing is used to approximate non-smooth components in the dynamics, thereby increasing the domain of convergence of the resulting TPBVPs. In the high-fidelity algorithm, Picard-Chebyshev is used for propagating the dynamics and the method of particular solutions is used for solving the TPBVPs. This combination affords avenues for increased efficiency without loss of accuracy. An example is presented to showcase each of the developed algorithms.

Time-Optimal (Earth to Lunar Quasi-Frozen Orbit)

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- Low-thrust time-optimal orbit transfers to a near-polar lunar quasi-frozen orbit (QFO) [4,7].
- Algorithm is formulated via indirect optimization methods, leading to a TPBVP.
- Split problem into 3 legs – 'Earth – Moon Sphere of Influence (SOI)', 'Moon SOI – Low Lunar Orbit (LLO)', 'LLO to QFO'.
- Sequential solution – Planar Cartesian & Polar, Spherical 3D, MEE 3D transfers and frame switch.
- Homotopy and arc-length continuation methods utilized to enhance convergence.

GEO to QFO:
Thrust: 0.5 N,
Isp: 3000 s,
Initial Mass: 250 kg.

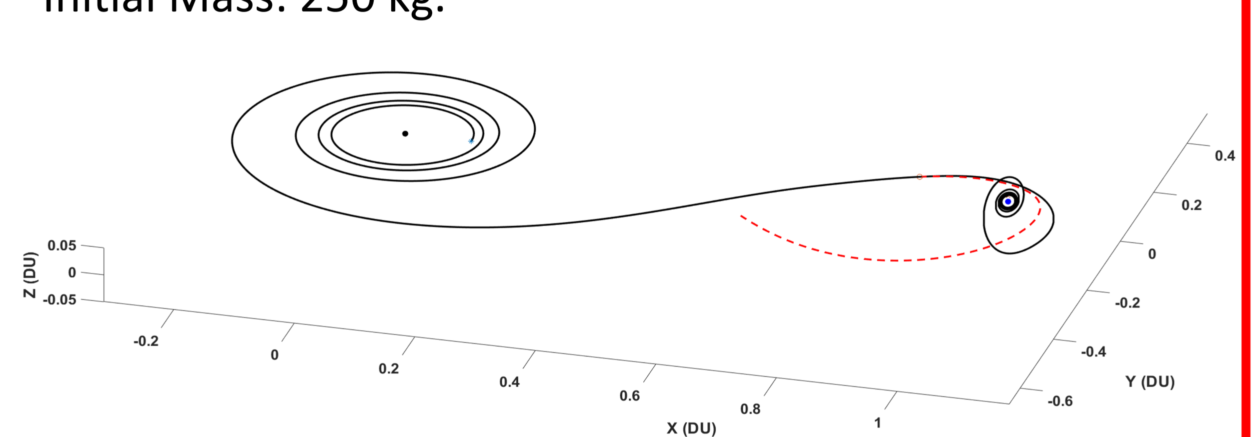

Figure 1: Low-thrust time-optimal orbit transfer from GEO to Polar LLO.

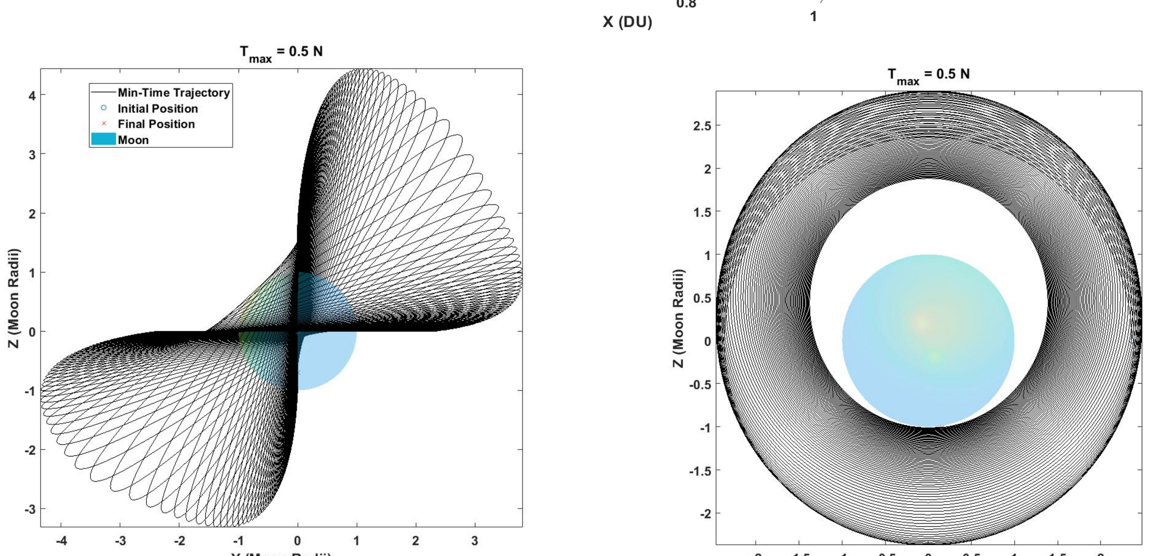

Figure 2: (left) Equatorial circular LLO to QFO (Case 2); (right) Polar circular LLO to QFO (Case 2) [4,7].

Trajectory Sequence	Total Transfer Time (days)	Number of Revs.		
		Leg 1	Leg 2	Leg 3
GEO – QFO via Equatorial LLO:	43.7	4	6	152
GEO – QFO via (i = 45°) LLO:	40.6	4	6	124
GEO – QFO via Polar LLO:	27.9	4	6	80

Fuel-Optimal (high-fidelity gravity model)

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- Low-thrust fuel-optimal orbit transfers with a high-fidelity (70x70) gravity model.
- Algorithm is formulated via indirect variational calculus approach, leading to a TPBVP.
- Method incorporates shadow constraints within the indirect optimization formulation.
- Constraints represent zero-thrust during eclipses.
- Non-smooth components are approximated with smooth representations using a hyperbolic tangent function and a continuation parameter.
- Dynamics propagated with the Picard-Chebyshev integrator and TPBVP is solved with the method of particular solutions (MPS).
- Computing the particular solutions with a reduced-fidelity force model greatly increases efficiency without loss of accuracy.
- GTO to GEO: initial mass = 100 kg, thrust magnitude is 0.5 N, specific impulse = 3100 s. The maneuver is completed over 6 days and makes 8 revolutions.

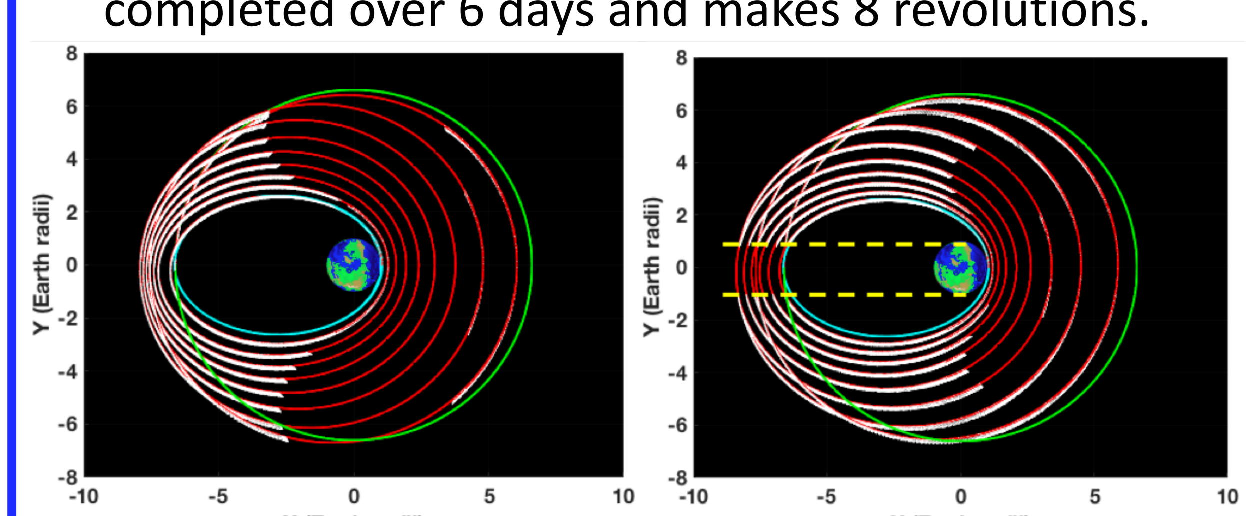

Figure 3: Low thrust fuel-optimal orbit transfer for the unconstrained (top left) and constrained (top right) problem.

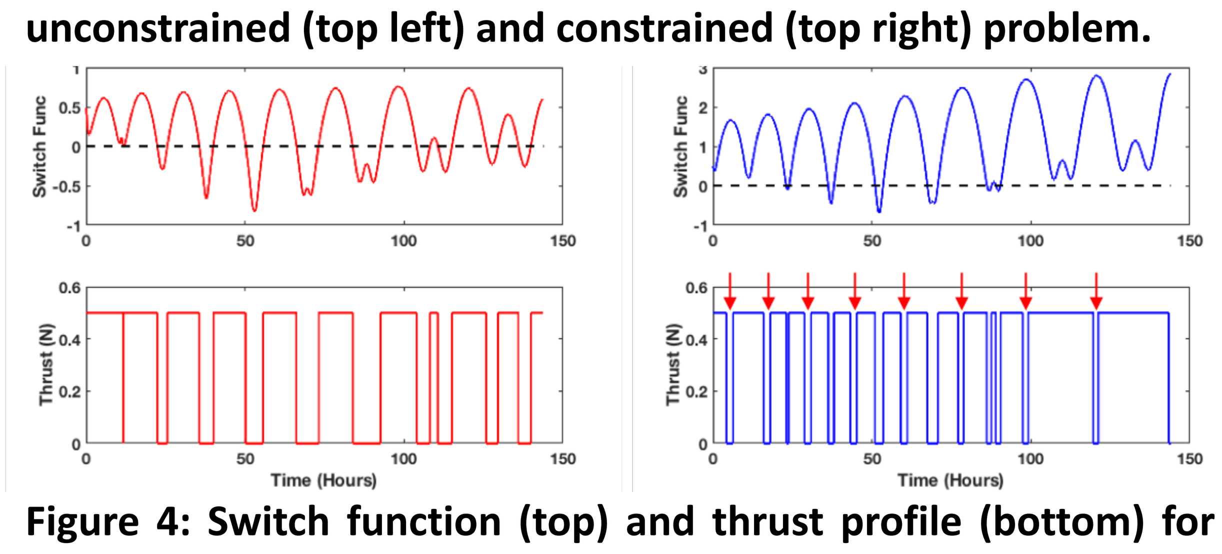

Figure 4: Switch function (top) and thrust profile (bottom) for the unconstrained (left) and constrained (right) problem. Red arrows indicate eclipse-induced zero-thrust arcs.

Hybrid Direct/Indirect (planetary flyby)

V. Arya, et al.

- Particle Swarm Optimization (PSO) outer-loop direct solver paired with optimal low-thrust fuel-optimal indirect inner-loop solver.
- Outer loop iterates on design variables: launch time, time-of-flight, b-plane angle, flyby periapsis ratio, v-infinity, and two angles.
- Indirect solver is formulated via variational calculus, leading to a TPBVP and a switch function.
- Thrust on/off switches are approximated using hyperbolic tangent smoothing and a continuation parameter.
- The global optimal solution and extremal map is identified by the PSO algorithm.

Earth-Venus-Mars:
Thrust = 0.75N,
Isp=2000s,
Initial mass= 1000 kg,
Time of flight = 478 days
Final mass: 649.5 kg

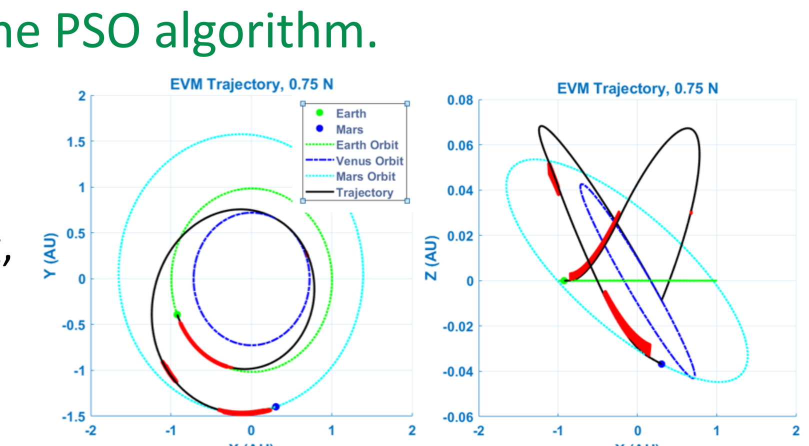

Figure 5: Low-thrust fuel-optimal orbit transfer (above) from Earth to Mars, with Venus flyby. Four thrust arcs and five coast arcs are present.

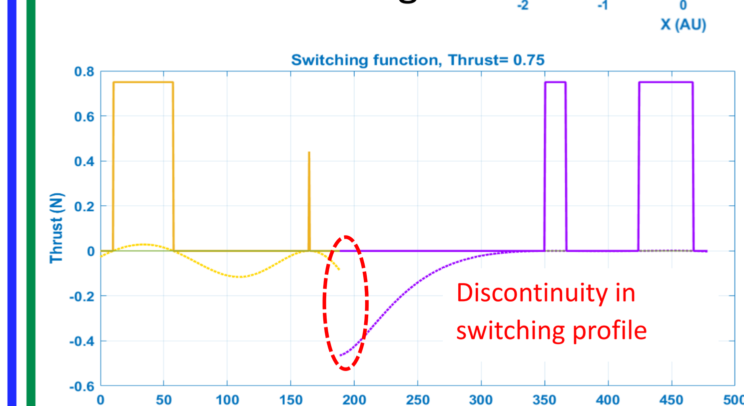

Figure 6: Switch function and thrust profile (above) of the optimal trajectory.

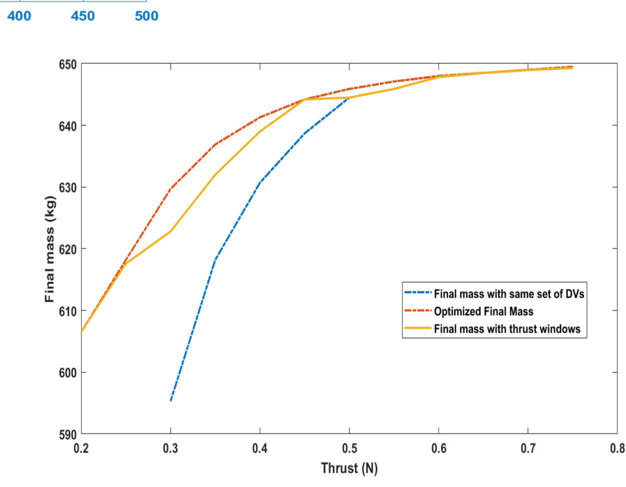

Figure7: Final mass trends (right) with continuation on thrust magnitude.

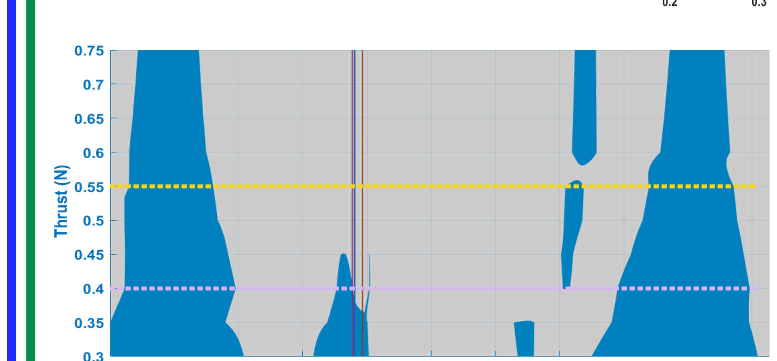

Figure 8: Switching surface contour plot (left) made via continuation of thrust magnitude.

Significance to NASA/JPL:

- Low thrust missions to the Moon, Mars and around the Earth, are of great interest to JPL and the general scientific community.
- Computing optimal low-thrust transfers with a high-fidelity gravity model is a challenging and computationally expensive process.
- We developed a technique that utilizes the Picard-Chebyshev propagator and MPS to accelerate convergence without loss of accuracy.
- We have also developed a method that strategically investigates large sets of candidate solutions to ensure a global optimal solution is found.

Publications

1. "Mission Design for Close-Range Lunar Mapping by Quasi-Frozen Orbits", S. Singh, E. Taheri, R. Woollands, J. Junkins, 70th International Astronautical Congress, Washington DC, Oct 2019.
2. "Gravity Assist Fuel-Optimal Low Thrust Trajectory Design using Hybrid Optimization Techniques", V. Arya, E. Taheri, R. Woollands, J. Junkins, 70th International Astronautical Congress, Washington DC, Oct 2019.
3. "Optimal Low-Thrust Gravity Perturbed Orbit Transfers with Shadow Constraints", R. Woollands, E. Taheri, Astrodynamics Specialist Conference, Portland, ME, Aug 2019.
4. "Feasibility of Quasi-Frozen, Near-Polar and Extremely Low-Altitude Lunar Orbits", S. Singh, R. Woollands, E. Taheri, J. Junkins, Acta Astronautica, submitted, Jul 2019.
5. "Efficient Computation of Optimal Low Thrust Perturbed Orbit Transfers", R. Woollands, E. Taheri, J. Junkins, Journal of Astronautical Sciences, May 2019.
6. "Efficient Computation of Optimal Low Thrust Perturbed Orbit Transfers", R. Woollands, E. Taheri, J. Junkins, 29th AAS/AIAA Space Flight Mechanics Conference, Maui, HI, Jan 2019.
7. "Feasibility of Quasi-Frozen, Near-Polar and Extremely Low-Altitude Lunar Orbits", S. Singh, R. Woollands, E. Taheri, J. Junkins, 29th AAS/AIAA Space Flight Mechanics Conference, Maui, HI, Jan 2019.

National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California
www.nasa.gov

PI/Task Mgr. Contact Info: robyn.m.woollands@nasa.jpl.gov

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