National Aeronautics and Space Administration

INHERENT INSTABILITIES IN SWARM FORMATIONS AND HOW TO AVOID THEM

Principal Investigator: Saptarshi Bandyopadhyay (347N)
Prof. Mark Balas, Vinod Gehlot (University of Tennessee Space Institute)

Program: Innovative Spontaneous Concept

Project Objective:
The objective of this spontaneous project was to develop a comprehensive framework for the analysis of N-dimensional geometric stability of multi-agent systems with general linear-time-invariant (LTI) and weakly nonlinear entities.

Benefits to NASA and JPL (or significance of results):
JPL, Caltech and NASA are increasing their research and technology development efforts in multi-agent systems and swarms for aerial and space applications. It is commonly assumed in the literature that the desired formation geometry/shape of these swarms does not affect the stability of the swarm. We showed that this assumption is incorrect, i.e., there exists desired formation geometries/shapes and naïve feedback control laws that can destabilize the entire swarm.

FY18/19 Results:

Result 1: Relative Error Dynamics and the Graph Laplacian
The following equation concisely captures the combined relative error dynamics of all the agents and the topology of their connectivity. This equation is the initial step in the stability analysis of multi-agent systems with linear dynamical entities.

Combined Dynamics

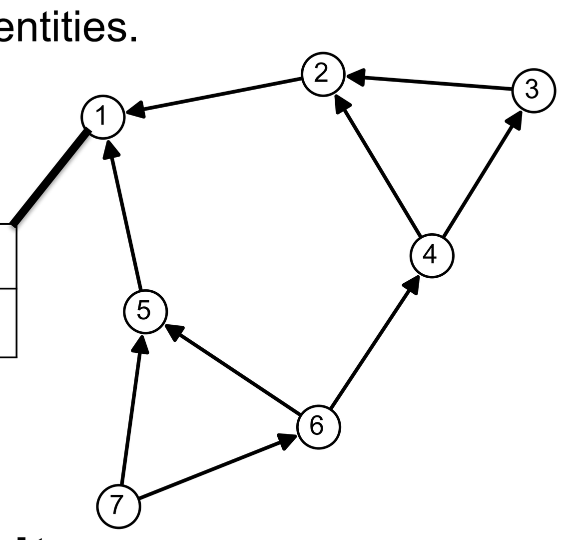
$$\dot{\xi} = (\hat{A} + \hat{B}(\mathrm{L} \otimes I_n)\hat{G}\hat{C})\xi + r = \tilde{A}\xi + r$$

Digraph Laplacian

Agent Dynamics
$\dot{x}_i = Ax_i + Bu_i$

Relative Error
$\xi_i \equiv \sum_{j \in N_i} x_i - x_j + r_{ij}$

Control Law
$u_i = GC\xi_i$



Result 2: Instability Under Isolated Reciprocity
A formation is said to have *reciprocity* if there exist bidirected edges between any two agents. In formations with isolated reciprocities, no two reciprocal connections are adjacent. Below are the two reciprocal connections analyzed in this project.

$$5 \rightleftarrows 4 \rightleftarrows 3 \rightarrow 2 \rightarrow 1$$

Tandem Reciprocity

$$5 \rightleftarrows 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$$

Single Reciprocity

Instabilities due to reciprocal connections can be inferred from the digraph Laplacian, the following example demonstrates this for tandem reciprocity.

$$\tilde{A} = \begin{bmatrix} A+BGC & -BGC & 0 & 0 & 0 \\ \boxed{-BGC} & A+2BGC & -BGC & 0 & 0 \\ 0 & \boxed{-BGC} & A+2BGC & -BGC & 0 \\ 0 & 0 & 0 & A+BGC & -BGC \\ 0 & 0 & 0 & 0 & A \end{bmatrix} \quad W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \boxed{\alpha_1} & 1 & 0 & 0 & 0 \\ \alpha_3 & \boxed{\alpha_2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Dynamic matrix for Tandem reciprocity — Transform matrix

The dynamic matrix shows that formation does not inherit stability from its entities; the matrix is not upper triangular. Create a coordinate transformation W to investigate the instability.

$$\mathrm{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Digraph Laplacian

Spectrum of L
$\sigma(\mathrm{L}) = \{\underbrace{3.247}_{\sigma_1}, \underbrace{0.198}_{\sigma_2}, \underbrace{0.155}_{\sigma_3}, 1, 0\}$

$$\bar{A} = \begin{bmatrix} A + \underbrace{p_1}_{=\sigma_1} BGC & -BGC & 0 & 0 & 0 \\ \underbrace{p_4}_{=0} BGC & A + \underbrace{p_2}_{=\sigma_2} BGC & -BGC & 0 & 0 \\ p_6 = 0 & p_5 = 0 & A + \underbrace{p_3}_{=\sigma_3} BGC & -BGC & 0 \\ 0 & 0 & 0 & A+BGC & -BGC \\ 0 & 0 & 0 & 0 & A \end{bmatrix}$$

$$\bar{A} = (W \otimes I_n)(\hat{A} + \hat{B}(\mathrm{L} \otimes I_n)\hat{G}\hat{C})(W^{-1} \otimes I_n)$$

Solve the polynomial equations p_4, p_5, and p_6 for α_i. In the new coordinates, the dynamic matrix will be upper triangular and the eigenvalues of the Laplacian matrix $\mathcal{L}$ scales the gain matrices, thereby directly affecting stability of the formation.

Result 3: A General Transformation to Investigate Stability
In complex formation networks, the transformations, as mentioned in Result 2, are challenging to obtain. We propose a general transformation to the eigen space of the digraph Laplacian, making the stability analysis of complex networks easier.

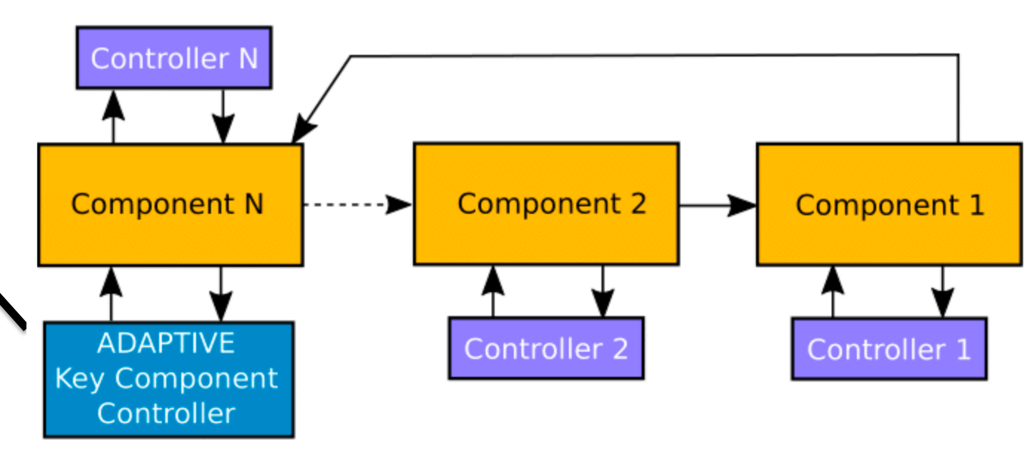
Theorem: *Let $\mathcal{L}$ be the Laplacian of a graph associated with the relative error system $\dot{\xi} = \tilde{A}\xi$. If $\mathcal{L}$ is diagonalizable, then there exists a coordinate transformation matrix $T = [\phi_1, \phi_2, \ldots, \phi_N]$, where $\phi_i \; \forall \; i \in I$ are the eigenvector of $\mathcal{L}$, such that the matrices $T \otimes I_n$ and $T^{-1} \otimes I_n$ diagonalizes the system matrix $\tilde{A}$ as follows,*

$$\begin{aligned} \bar{A} &= (T \otimes I_n)(\hat{A} + \hat{B}(\mathrm{L} \otimes I_n)\hat{G}\hat{C})(T^{-1} \otimes I_n) \\ &= diag(A + B(\sigma_1 G)C, \ldots, A + B(\sigma_N G)C) \end{aligned}$$

Result 4: Restore Stability using Adaptive Key Components
Many times, in complex formation network topologies, it can be difficult to find fixed output feedback gains to guarantee stable error dynamics. We introduce the concept of an adaptive-key-components to restore stability in these circumstances.

Adaptive Gain
$\dot{G}_e = -yy^T\gamma_e - aG_e;$ $\gamma_e, a > 0$

Kalman Yacubovic Conditions (ASD)
$A^T P + PA = -Q$ $PB = C^T; \quad P, Q > 0$



Any or all the agents in the formation can be adaptive key components, provided, that the relative error dynamics are almost strictly dissipative (ASD). The following result shows the ASD property for a formation.

Theorem: The relative error dynamics given by $\dot{\xi} = \tilde{A}\xi + r = (\hat{A} + \hat{B}(\mathrm{L} \otimes I_n)\hat{C})\xi + r$, is ASD if and only if the triple $(\hat{A}, \hat{B}, \hat{C})$ is ASD.

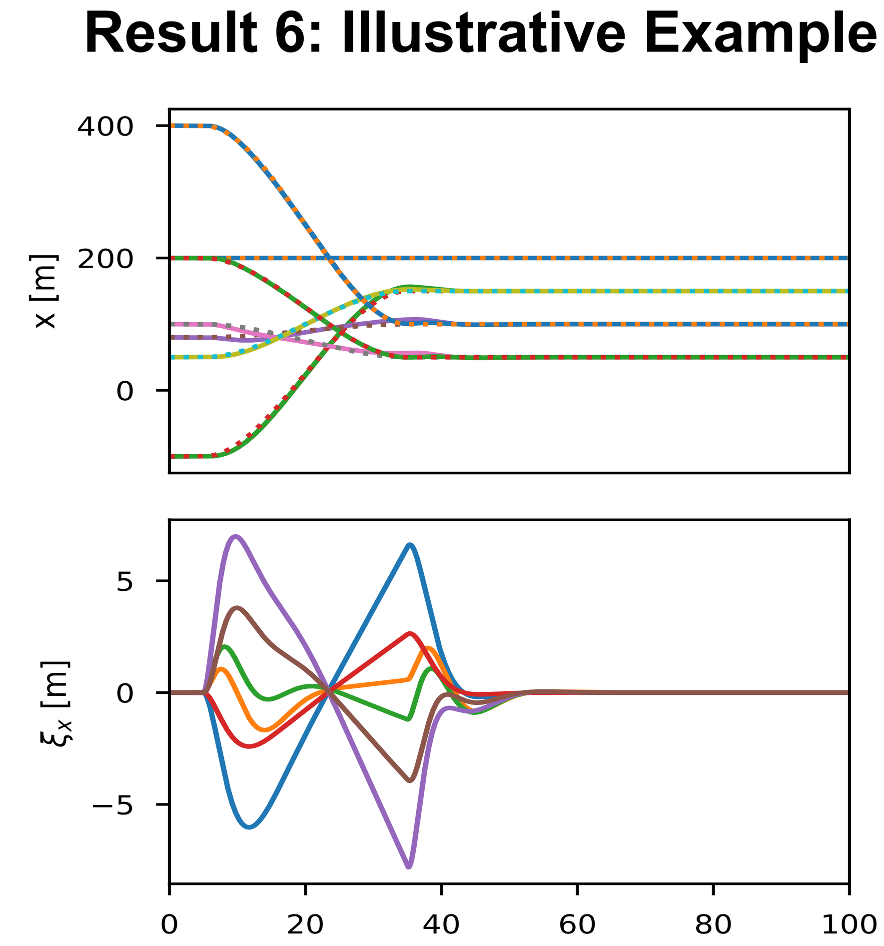
Result 5: Instabilities due to Weak Nonlinearities
Perturbations due to a planet's oblateness, atmospheric drag, gravitational potential variations, etc., induces weak nonlinearities into the dynamics of agents. The equation below show the model for weakly nonlinear entities:

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + \varepsilon h(x_i) \\ y_i &= C(\mathrm{L}^{(i)} \otimes I_n)x_i + CR_{ij} \end{aligned}$$

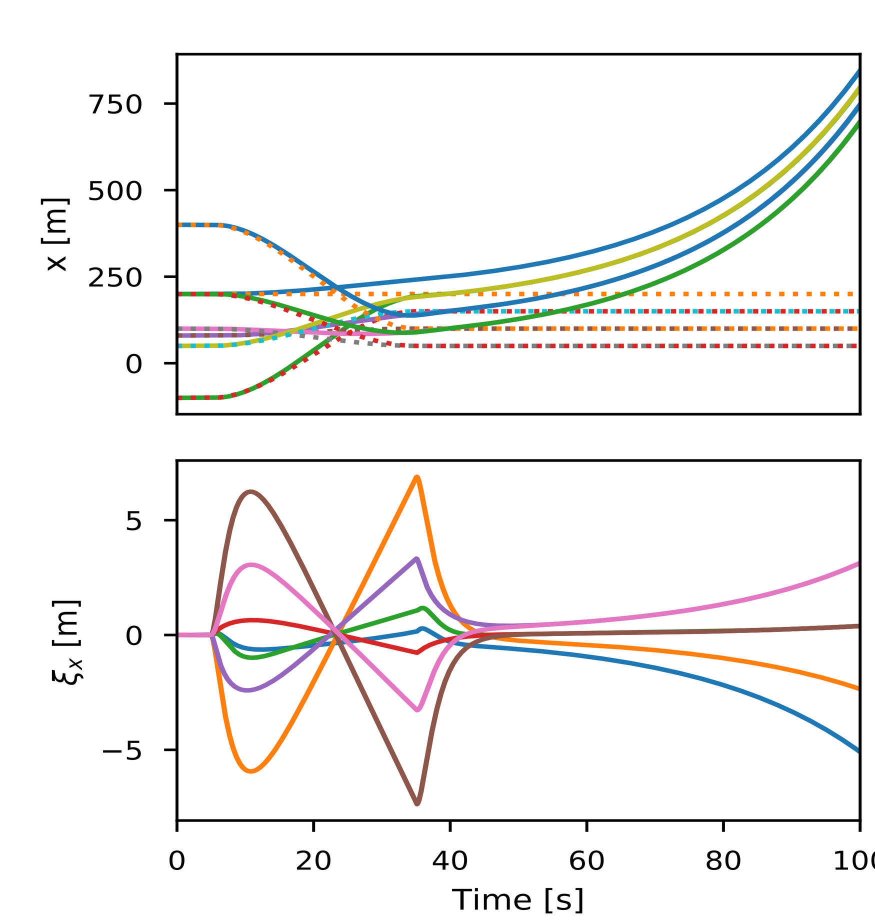
We introduce the concept of ideal trajectories to simplify the analysis of formation stability with nonlinear entities, and proved three theoretical results that show:

- *Weak nonlinearities reduce the stability margin of swarms, regardless of the network topology defined by the digraph Laplacian.*
- *The ASD property for the entire swarm is preserved with weak nonlinearities.*
- *Assuming the nonlinearities are "weak," the adaptive key component can be used to restore stability.*

Result 6: Illustrative Example



V-Formation with no reciprocity showing stable behavior.



V-Formation with reciprocity showing unstable behavior.

National Aeronautics and Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

www.nasa.gov

Publications:

V. P. Gehlot, M. J. Balas and S. Bandyopadhyay, "(Accepted) Dynamic Stability And Adaptive Control of Networked Evolving Formations with Weak Nonlinearities," in AIAA SciTech Forum and Exposition, Orlando, 2020.

PI/Task Mgr. Contact Information:
Saptarshi.Bandyopadhyay@jpl.nasa.gov, 6263184174

Poster No. 47