# PARALLEL ORBIT PROPAGATION TECHNIQUES

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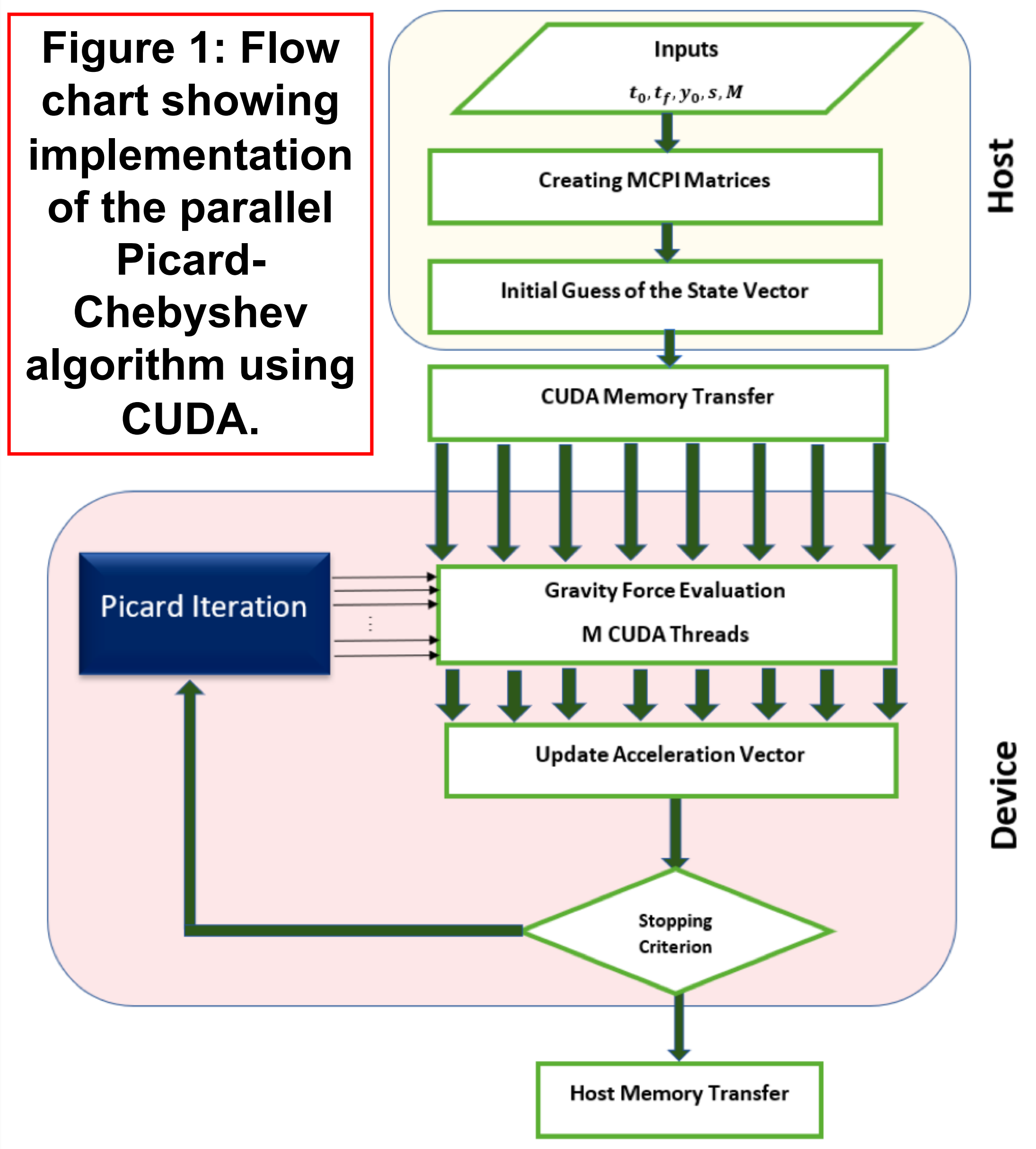
**Program: Spontaneous R&TD**

**Abstract:** We have developed a parallel Picard-Chebyshev propagator for fast and accurate orbit propagation through a high-fidelity gravity field. Current orbit propagators used by the JPL are inherently serial, stepping forward in time from an initial condition and applying forces based on the previous state. The Picard-Chebyshev propagator is a path approximation numerical integrator, that is, long state trajectory arcs are approximated continuously and updated at all instances on each iteration. In serial mode, the Picard-Chebyshev method has been shown to outperform a 12th order Runge-Kutta integrator and an 8th Gauss-Jackson integrator [1]. The Picard path iteration approach is fundamentally parallel, as forces at multiple time steps can be evaluated simultaneously. We show results for parallel and serial Earth orbit propagation through a 200x200 spherical harmonic gravity field. In addition, we developed a parallel approach for evaluating spherical harmonic gravity and a parallel approach for evaluating a finite element approximation of the gravity model.

## Parallel Implementation

The Picard-Chebyshev integrator is parallelized using C++ and CUDA. Parallelization is done at the acceleration level (Figure 1) for both the spherical harmonic gravity model (SHM) and the finite element model (FEM). That is, the gravitational acceleration at each Chebyshev node along the trajectory is computed simultaneously. We present results for propagating four Earth orbiting satellites shown in Figure 2: low Earth orbit (LEO), geostationary orbit (GEO), medium Earth orbit (MEO), high Earth orbit (HEO). In Figure 3 we show the computational efficiency results. As expected, the parallel algorithms are more efficient than their serial counterparts. The parallel FEM also requires less computation time than the serial and parallel SHM. However, this comes at a price of reduced accuracy, 10 digits for the FEM versus 14 digits for the SHM.

**Figure 1: Flow chart showing implementation of the parallel Picard-Chebyshev algorithm using CUDA.**


Host
Inputs
$t_0, t_f, y_0, s, M$
Creating MCPI Matrices
Initial Guess of the State Vector
CUDA Memory Transfer
Device
Picard Iteration
Gravity Force Evaluation
M CUDA Threads
Update Acceleration Vector
Stopping Criterion
Host Memory Transfer

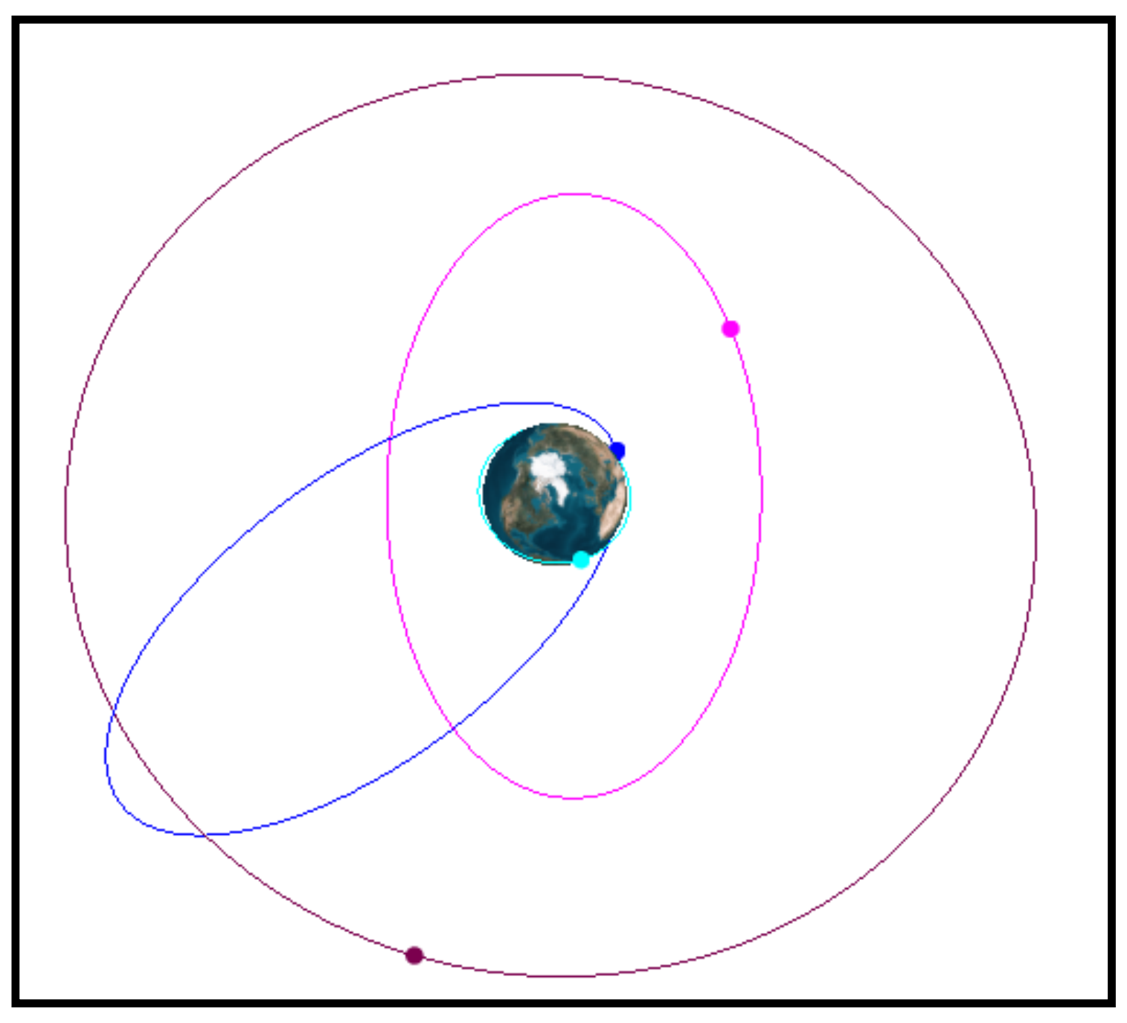


**Figure 2: The four orbit test cases utilized for testing the algorithm.**

**Figure 3: Computational comparison in serial and parallel, using SHM and FEM gravity models, for the four orbit test cases.**

## Finite Element Gravity Model

We use classical discrete polynomial approximation to solve for a piecewise approximation of high-degree and order gravity models. We replace the higher order perturbations in the EGM2008 spherical harmonic model by a finite element model orthogonal approximation [2,3]. Thus, much lower degree, locally valid, functions can be used to efficiently model and compute local gravity perturbations. The total gravity potential is split into a reference and disturbance term. The reference includes two-body plus J2, and the higher order gravity terms are considered perturbative. The FEM is developed from 2D mesh grid covering a sphere of specified radius. A large family of spherical shells is sampled using a cosine distribution in the radial direction. For each 2D cell on a particular shell, gravity is approximated using a 2D Chebyshev polynomial. A 16th degree polynomial is used in Region I and a 13th degree polynomial in Region II (Figure 4). We have also structured the FEM models to render them radially adaptive, thus further reducing computational cost. Figure 5 shows a run-time comparison for each of the algorithms as a function of the number of Chebyshev nodes used in the approximation.

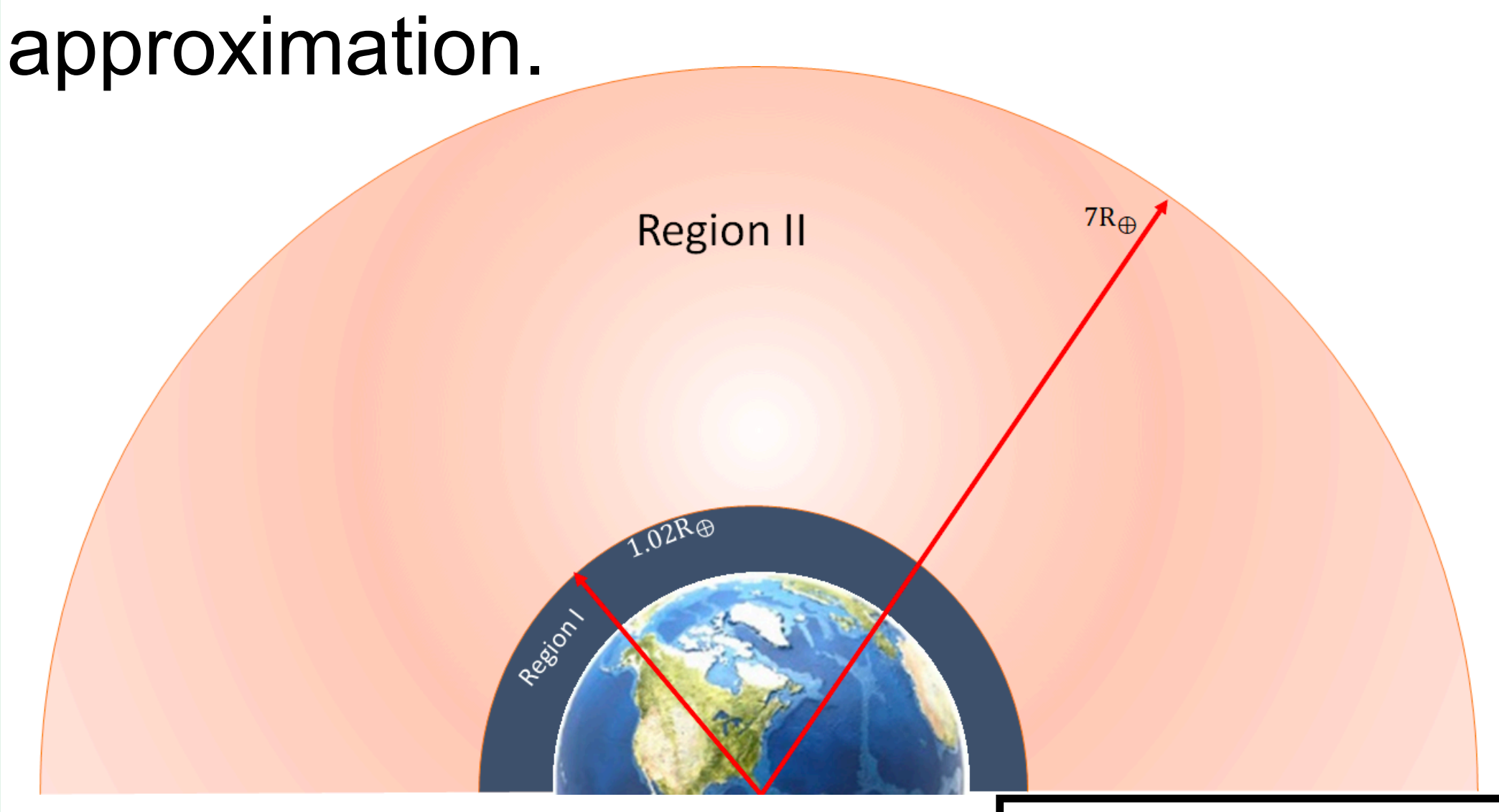
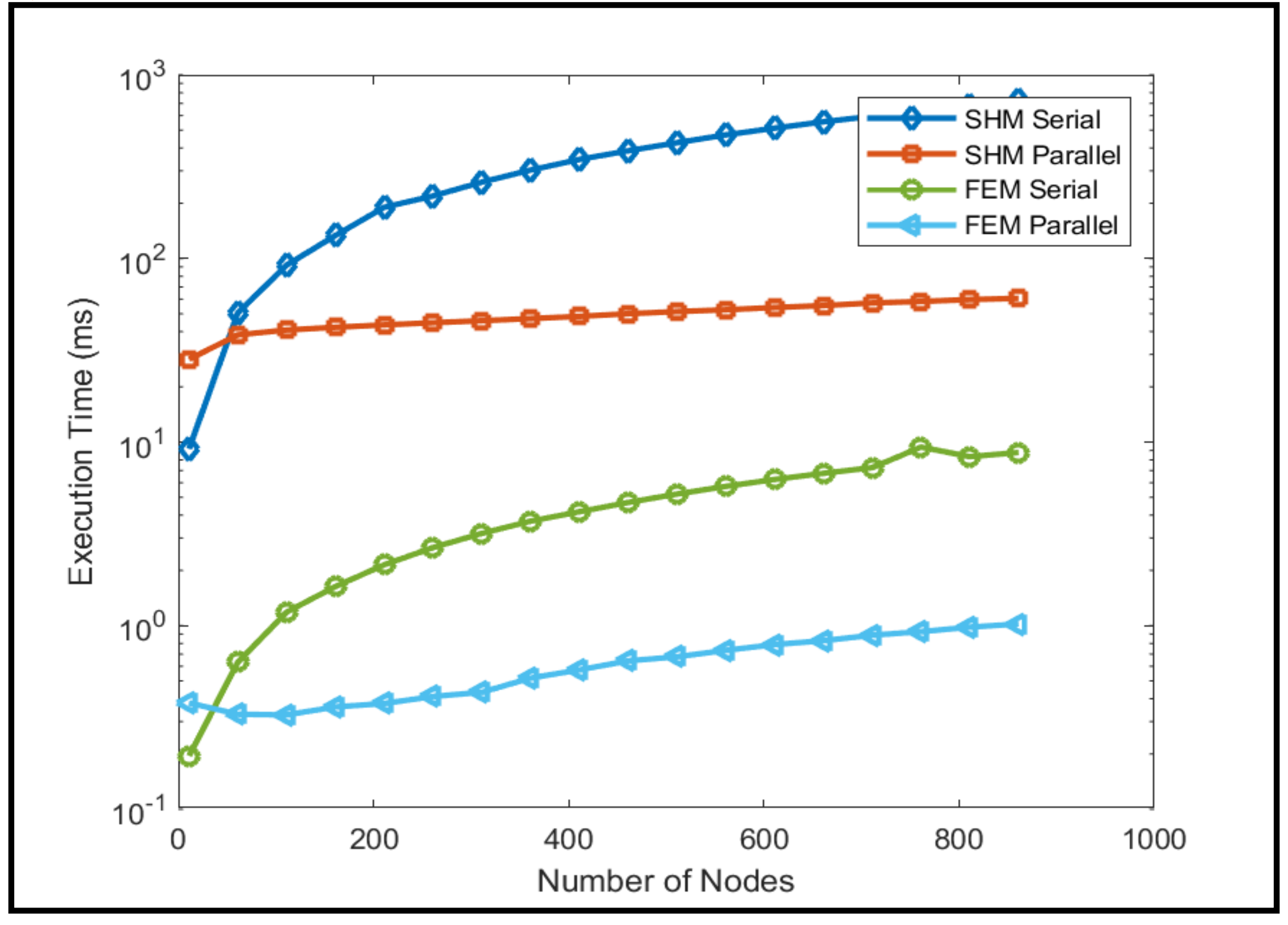


**Figure 4: Schematic showing the two different FEM gravity approximations, a 16th degree Chebyshev polynomial in Region I and a 13th degree Chebyshev polynomial in Region II.**

**Figure 5: Run-time comparison for SHM and FEM gravity models, for a varying number of nodes used in the Chebyshev polynomial approximation.**



## Benefits to NASA/JPL

The parallel Picard-Chebyshev propagator could be integrated with existing MONTE software for fast and accurate mission planning and spacecraft operations. It would be immediately useful to a number of spacecraft requiring high-fidelity gravity models, such as the Earth/Mars/Lunar orbiters. Having a radially adaptive FEM in a polynomial approximation framework allows higher-order state transition tensors to be easily accessed, which is useful for gravity parameter estimation in the orbit determination process.

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**References:**
[1] Woollands, R., Junkins, J., “Nonlinear Differential Equation Solvers via Adaptive Picard-Chebyshev Iteration: Applications in Astrodynamics”, JGCD, Jan 2019.
[2] Bani Younes, A., “Orthogonal Polynomial Approximations in Higher Dimensions: Applications in Astrodynamics”, PhD Dissertation, Texas A&M University, Aug 2013.
[3] Junkins, J., Bani Younes, A., Woollands, R., Bai, X., “Efficient and Adaptive Orthogonal Finite Element Representation of the Geopotential, JAS, Jun 2017.

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