National Aeronautics and Space Administration

Trajectory optimization with uncertainties using stochastic optimal control

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Program: Spontaneous Concept

Project Objective :

- The objective is to **introduce uncertainties in the design of deep-space reference trajectories**.
- Typically, reference trajectories are solutions of deterministic, nonlinear, highly-dimensional optimization problems with multiple objectives. Robustness is evaluated a-posteriori as a product of the navigation analysis, and if needed, the reference trajectory is corrected in an iterative process that can take days or weeks to converge and lead to conservative solutions.
- Modeling uncertainties in the design is not current practice in our section - although it is common in other fields, even at JPL such as robotic path planning, using the mathematical framework of stochastic optimal control or robust control.
- This project shows that uncertainties can and should be modeled also in the reference trajectory optimization. We identify key stochastic problems in astrodynamics and some novel approaches to solve them

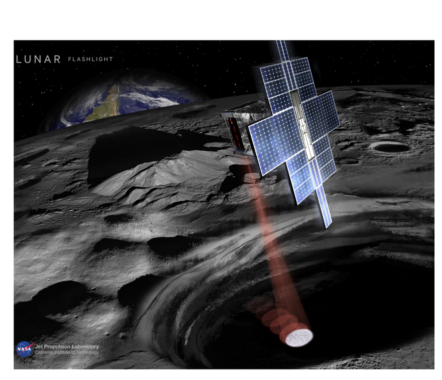
FY 19 Results (summary):

We identify problems in astrodynamics that would benefit from a stochastic optimization, discuss the current state of the art, and in some cases develop new mathematical formulation and algorithms (that will be presented in upcoming technical conferences). The problems affect a variety of missions:

- **Low-thrust SEP missions**, which need to be robust to thrust outages called Missed Thrust Events. *See separate box on Missed Thrust Events*
- **Planetary satellite exploration missions**, which are enabled by tours subject to probability constraints and driven by stochastic DVs. *See separate box on Satellite Tours. Details to appear in [1].*
- **Deep-space MicroSats and CubeSats**, which are enabled by trajectories robust to large execution errors and poor knowledge, with sparse opportunities for uploading trajectory correction maneuvers. The full optimization of the reference trajectory should include stochastic DV costs and model uncertainties, OD campaign, and the controller, to automatically provide the solution that meets the mission objective with the minimum propellant and with a desired probability of success. Specific examples are:
  - Design of low-thrust transfers robust to large knowledge and execution errors [2]
  - Design of the unstable orbits around the Moon that minimize the station keeping or formation flying costs (open loop optimization that includes Kalman filters and a closed loop controller) [3]
  - Design of semi-hard lunar lander trajectories, where the approach trajectory is optimized to increase the probability of success [4]

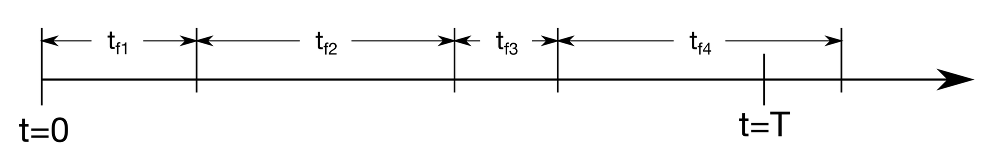
Benefits to NASA and JPL:

The trajectory design is a critical part of any deep-space exploration missions. Modeling uncertainties in the reference trajectory optimization would greatly improve the design process for missions such as: Clipper and any other planet mission that involve satellite flybys (Enceladus, Uranus, Neptune/Triton); any low-thrust missions; any lunar or deep-space MicroSat or CubeSat mission.



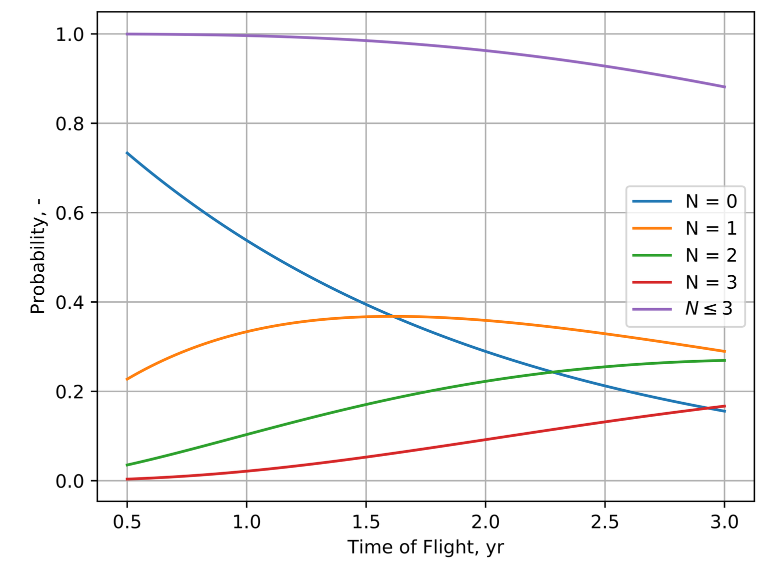
Missed Thrust Events (MTEs) for low-thrust SEP missions:

- MTEs are thrust outages typically caused by safe modes, that results in the spacecraft drifting away from its nominal reference trajectory. Recovery is expensive: multiple outages could result in the loss of science goals or even the entire mission.
- To incorporate MTEs into trajectory design, we need statistical information on MTEs. We know that the time between MTEs is modeled by a Weibull [5,6].
- For a mission duration T, the probability of n events is a derived distribution, which is well approximated by an Erlang distribution.



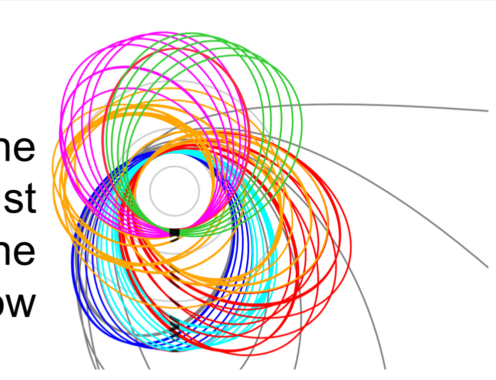
- The probability of failure for a mission of duration T is computed as

$$P(\text{fail}) = P(\text{fail}|n=0)P(n=0) + \\ + P(\text{fail}|n=1)P(n=1) + \\ + P(\text{fail}|n=2)P(n=2) + \ldots$$

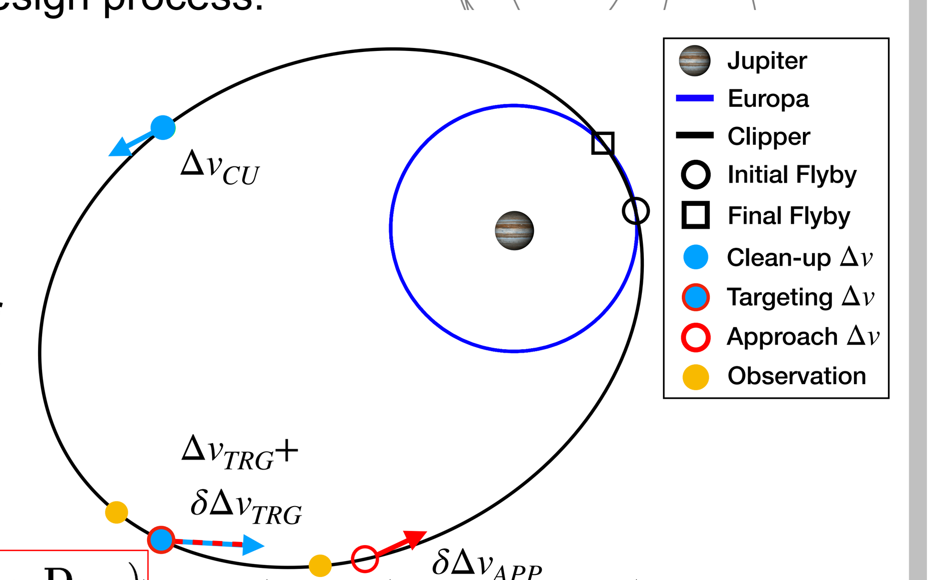


Satellite Tours

Tours are spacecraft trajectories with tens of revolutions and flybys, used for the exploration of natural satellites. In such dynamics, uncertainties grow quickly and must be accounted for in the design of the reference trajectory. In the current approach, the effect of uncertainties is evaluated a posteriori; this research explains how uncertainties can instead be taken into account in the trajectory design process.



Consider one single leg of a Europa Clipper tour – from one Europa flyby to the next. The initial and final states are fixed. The initial states, the DV maneuvers, and the range and range-rate measurements are affected by uncertainties. ***What deterministic maneuvers should we plan, that minimize the total DV (including that for statistical correction) of 99% of the realizations, and meet a probability of impact constraint at Europa of 1e-3?*** We formulate the problem as:

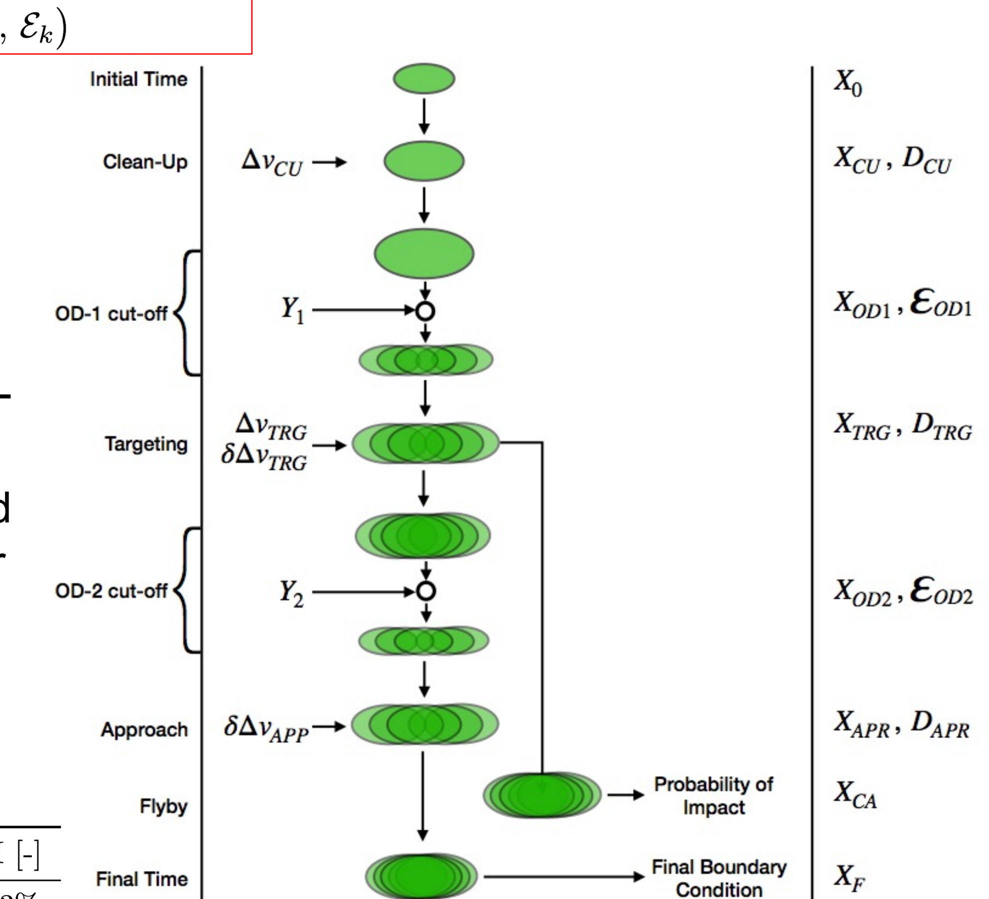


$$\begin{aligned} \min_{\Delta \mathbf{v}_{\mathrm{CU}},\Delta \mathbf{v}_{\mathrm{TRG}}} \quad & \mathcal{Q}_{\Delta V_{\mathrm{TOT}}}(0.99) \\ \text{s.t.} \quad & \mathbf{X}_k = \mathcal{T}(\mathbf{X}_{k-1}, \Delta\mathbf{v}_{k-1}, \mathbf{D}_{k-1}, \mathcal{E}_k) \\ & \mathbf{X}_0 \sim \mathcal{N}_{\mathbf{X}_0}(\mathbf{m}_0, \mathbf{P}_0) \\ & Pr(R_{CA} \leq R_{EUR}) \leq 1e\text{-}3 \\ & \mathbb{E}[\mathbf{R}_f] = \bar{\mathbf{r}}_f \ , \end{aligned}$$

$$\mathbf{X}_k^- = \mathcal{T}_{\text{predict}}(\mathbf{X}_{k-1}^+, \mathbf{u}_{k-1}, \mathbf{D}_{k-1})$$
$$\mathbf{X}_k^+ = \mathcal{T}_{\text{update}}(\mathbf{X}_k^-, \mathcal{E}_k)$$

The transcription of the stochastic process τ includes:

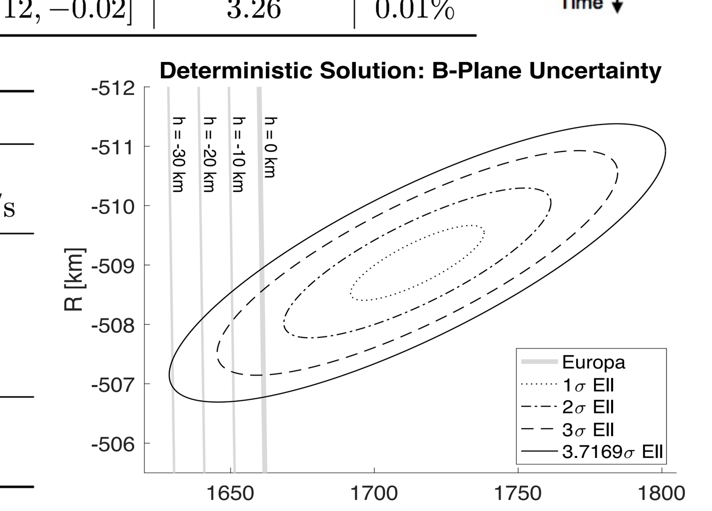
- Nonlinear Uncertainty Propagation $\tau_{predict}$: Chapman-Kolgomorov eq. by Sparse Gauss-Hermite quadrature
- Nonlinear Uncertainty Update τ_{update} : observations and Bayes' rule with Sparse Gauss-Hermite Quadrature Filter
- K-inverse controller;
- Objective and Constraint from propagated distributions

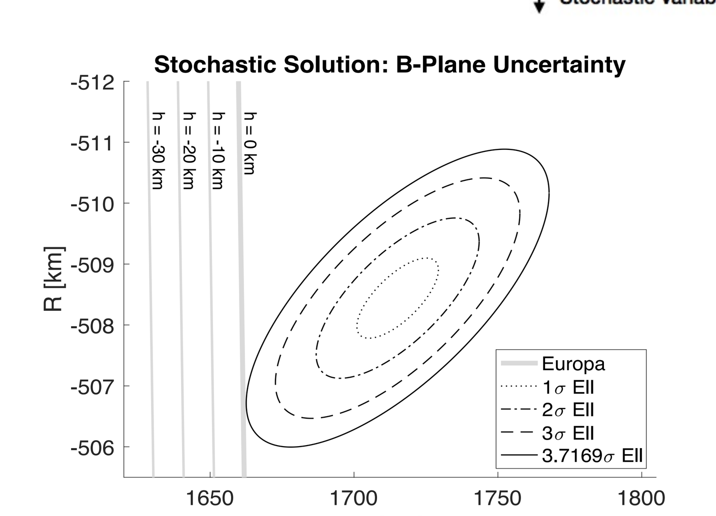


Numerical example

Solution	ΔV_{CU} [m/s]	ΔV_{TRG} [m/s]	$\Delta V99$ [m/s]	PoI [-]
Deterministic	[0.0, 0.0, 0.0]	[−1.30, +2.86, +3.23]	4.54	1.03%
Stochastic	[+0.09, +0.19, +0.24]	[−1.98, +2.12, −0.02]	3.26	0.01%

Uncertainty	Component	Value
Initial Dispersion	Position (RTN)	[3.7, 5.3, 9.3] m
	Velocity (RTN)	[2.3, 0.3, 0.5] mm/s
Execution Error (Gates Model)	Fixed Pointing	3.33 mm/s
	Prop. Pointing	6.67 mrad
	Fixed Magnitude	4.67 mm/s
	Prop. Magnitude	3.33e-3
Observation Accuracy	Range	10.0 m
	Range-rate	5.0 mm/s





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Publication:

[1]Greco, Campagnola, Vasile, "Robust Space Trajectory Design using Belief Stochastic Optimal Control", SFMM 2020 (to appear)

References:

[2] Ozaki, Campagnola, Funase, Tube Stochastic Optimal Control For Non Linear Constrained Trajectory Optimization Problems, 2019 SFMM

[3] Oguri, Oshima, Campagnola, Kakihara, Ozaki, Baresi, Kawakatsu, Funase, Ryu, EQUULEUS Trajectory Design, JAS (accepted)

[4] Hernando-Ayuso, Campagnola, Yamaguchi, Ozawa, Ikenaga, OMOTENASHI trajectory analysis and design: Landing phase, AA, 156, (2019) 113-124

[5] Imken et al., "Modeling spacecraft safe mode events", IEEE Aerospace Conf., 2018

[6] Laipert and Imken. A Monte Carlo approach to measuring trajectory performance subject to missed thrust. SFMM 2018

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