National Aeronautics and Space Administration

Dynamics Modeling of Articulated Space Robotic Spacecraft Using Dual Quaternions

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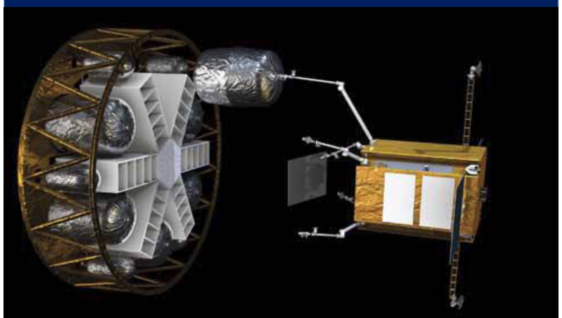
Program: SURP

PROJECT OBJECTIVE:

To provide a unified framework enabling rigid body kinematic and dynamic analysis and control of serial manipulators mounted on spacecraft via dual quaternions (DQ)

- **Extend DQ theory to the N-Body spacecraft-manipulator system problem with arbitrary tree structure (i.e., an arbitrary number of links or robotic arms on the spacecraft)**
- **Develop control strategies for spacecraft-manipulator systems in DQ framework**
- **Code and simulate N-Body spacecraft-manipulator system performing rendezvous and proximity operations**



BACKGROUND:

Modeling and simulation of rigid multibody systems has been a topic of interest since the 70's. Naturally, the translation and rotation of bodies is coupled, but mathematical formulations usually separate the two types of motions. Dual quaternions by construction capture such coupling. In the past, this has been mainly used for fixed-base kinematic tasks. In contrast, this research aims to incorporate the natural coupling for capturing the complex dynamics and control of a robotic manipulator operating on a free-floating base satellite.

BENEFITS TO NASA AND JPL:

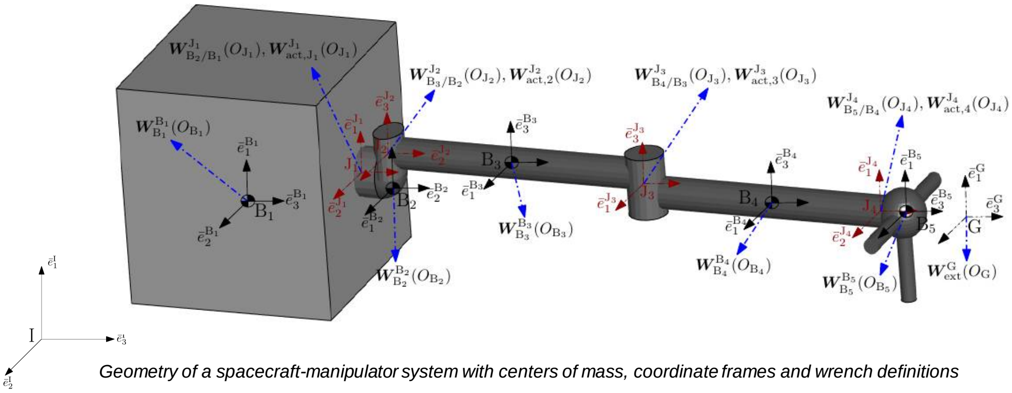
Research provides a compact and closed-form mathematical framework to perform robotic servicing of orbiting spacecraft

- **Servicing includes tasks such as visual inspections, scheduled maintenance, refueling, part replacement, repair of worn or broken components, or completion of failed deployment sequences**
- **Dual quaternion-based robotics tools enable servicing tasks**
- **Simplifies the specification of complicated constraints into the control design**

NEW TECHNOLOGY REPORTING:

NTR 50644, Dual Quaternions for Dynamics & Control of Spacecraft with Robotic Arms

Dual Quaternion Modeling of Spacecraft-Manipulator Systems



Geometry of a spacecraft-manipulator system with centers of mass, coordinate frames and wrench definitions

State Space Representation of System:

$$x = \begin{bmatrix} \boldsymbol{q}_{\oplus_1/\mathrm{I}} \\ \Gamma \\ \boldsymbol{\upsilon} \end{bmatrix} \in \mathbb{H}_d \times \mathbb{R}^6 \times \mathbb{H}_d^v \times \mathbb{H}_d^v \times \mathbb{H}_d^v \times \mathbb{H}_d^v \times \mathbb{H}_d^v,$$

where $\boldsymbol{q}_{\oplus_1/\mathrm{I}} \in \mathbb{H}_d$ is the pose of the base, and where

$$\Gamma = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \Gamma_3 & \Gamma_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^6,$$

are the generalized coordinates given by,

$$\Gamma_1 = \theta_{\mathrm{J}_1/\oplus_1},\ \Gamma_2 = \theta_{\mathrm{J}_2/\oplus_2},\ \Gamma_3 = \theta_{\mathrm{J}_3/\oplus_3},\ \text{and}$$
$$\Gamma_4 = [\phi_{\mathrm{J}_4/\oplus_4},\ \theta_{\mathrm{J}_4/\oplus_4},\ \psi_{\mathrm{J}_4/\oplus_4}]^{\mathrm{T}}.$$

The dual velocities $\boldsymbol{\upsilon} \in \mathbb{H}_d^v$ of the system are given by

$$\boldsymbol{\upsilon} = \begin{bmatrix} \boldsymbol{\omega}^{\oplus_1}_{\oplus_1/\mathrm{I}}, & \boldsymbol{\omega}^{\oplus_2}_{\oplus_2/\mathrm{I}}, & \boldsymbol{\omega}^{\oplus_3}_{\oplus_3/\mathrm{I}}, & \boldsymbol{\omega}^{\oplus_4}_{\oplus_4/\mathrm{I}}, & \boldsymbol{\omega}^{\oplus_5}_{\oplus_5/\mathrm{I}} \end{bmatrix}^{\mathrm{T}}.$$

Time Evolution of Kinematics:

The pose of the satellite base evolves as

$$\dot{\boldsymbol{q}}_{\oplus_1/\mathrm{I}} = \frac{1}{2}\boldsymbol{q}_{\oplus_1/\mathrm{I}}\boldsymbol{\omega}^{\oplus_1}_{\oplus_1/\mathrm{I}}.$$

The generalized speed for each joint coordinate are

$$\dot{\Gamma}_i = Z_{i,\mathrm{R}} \star \boldsymbol{\omega}^{\mathrm{J}_i}_{\mathrm{J}_i/\oplus_i} = Z_{i,\mathrm{R}} \star \left(\boldsymbol{q}_{\oplus_{i+1}/\mathrm{J}_i}\boldsymbol{\omega}^{\oplus_{i+1}}_{\oplus_{i+1}/\mathrm{I}}\boldsymbol{q}^*_{\oplus_{i+1}/\mathrm{J}_i} - \boldsymbol{q}^*_{\mathrm{J}_i/\oplus_i}\boldsymbol{\omega}^{\oplus_i}_{\oplus_i/\mathrm{I}}\boldsymbol{q}_{\mathrm{J}_i/\oplus_i}\right),$$

where $Z_{i,\mathrm{R}}$ is the screw matrix for joint $i \in \{1, \dots, \mathrm{N}-1\}$.

Time Evolution of Dynamics:

General form of system of equations for dynamics:

$$\begin{bmatrix} \mathcal{S}_{11} & \mathcal{S}_{12} \\ \mathcal{S}_{21} & \mathcal{S}_{22} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\upsilon}} \\ \mathcal{T} \end{bmatrix} = \begin{bmatrix} \mathcal{B}_1 \\ \mathcal{B}_2 \end{bmatrix},$$

where $\mathcal{S}_{11} = \mathrm{diag}\left(\mathrm{H}(M_{\oplus_1}), \mathrm{H}(M_{\oplus_2}), \mathrm{H}(M_{\oplus_3}), \mathrm{H}(M_{\oplus_4}), \mathrm{H}(M_{\oplus_5})\right) \in \mathbb{R}^{40\times 40}$,
$\mathcal{S}_{22} = 0_{18\times 18}$, $\mathcal{T} = \left[(\tilde{\boldsymbol{W}}^{\mathrm{J}_1}_{\oplus_2/\oplus_1}(O_{\mathrm{J}_1}))^{\mathrm{T}},\ (\tilde{\boldsymbol{W}}^{\mathrm{J}_3}_{\oplus_3/\oplus_2}(O_{\mathrm{J}_2}))^{\mathrm{T}},\ (\tilde{\boldsymbol{W}}^{\mathrm{J}_3}_{\oplus_4/\oplus_3}(O_{\mathrm{J}_3}))^{\mathrm{T}},\ (\tilde{\boldsymbol{W}}^{\mathrm{J}_4}_{\oplus_5/\oplus_4}(O_{\mathrm{J}_4}))^{\mathrm{T}}\right]^{\mathrm{T}}$.
For $i \in \{1, \dots, \mathrm{N}-1\}$ and for $j \in \{1, \dots, \mathrm{N}\}$,

$$(\mathcal{S}_{12})_{ji} = \begin{cases} -\mathrm{Q}(\boldsymbol{q}^*_{\oplus_j/\mathrm{J}_i})_{\mathrm{L}}V_i, \\ 0_{8\times r_i}, \\ \mathrm{Q}(\boldsymbol{q}^*_{\mathrm{J}_i/\oplus_j})_{\mathrm{R}}V_i, \end{cases} \quad (\mathcal{S}_{21})_{ij} = \begin{cases} \Lambda_i\mathrm{Q}(\boldsymbol{q}^*_{\oplus_j/\mathrm{J}_i})_{\mathrm{R}}, & \text{if joint } i \text{ is proximal, body } j \text{ is distal,} \\ 0_{r_i\times 8}, & \text{if joint } i \text{ is not connected to body } j, \\ -\Lambda_i\mathrm{Q}(\boldsymbol{q}^*_{\mathrm{J}_i/\oplus_j})_{\mathrm{L}}, & \text{if joint } i \text{ is distal, body } j \text{ is proximal,} \end{cases}$$

where $\mathrm{Q}(\cdot)_{\mathrm{L}}$, $\mathrm{Q}(\cdot)_{\mathrm{R}}$, Λ_i, and V_i are mapping matrices.
Vector $\mathcal{B}_1$ is associated with r.h.s. of Newton-Euler EOM,

$$(\mathcal{B}_1)_1 = -\boldsymbol{\omega}^{\oplus_1}_{\oplus_1/\mathrm{I}} \times \left(M_{\oplus_1} \star \left((\boldsymbol{\omega}^{\oplus_1}_{\oplus_1/\mathrm{I}})\right)^{\mathrm{s}}\right) + \boldsymbol{W}^{\oplus_1}_{\oplus_1}(O_{\oplus_1}) - \boldsymbol{q}_{\mathrm{J}_1/\oplus_1}\boldsymbol{W}^{\mathrm{J}_1}_{\mathrm{act,J}_1}(O_{\mathrm{J}_1})\boldsymbol{q}^*_{\mathrm{J}_1/\oplus_1},$$
$$(\mathcal{B}_1)_2 = -\boldsymbol{\omega}^{\oplus_2}_{\oplus_2/\mathrm{I}} \times \left(M_{\oplus_2} \star \left((\boldsymbol{\omega}^{\oplus_2}_{\oplus_2/\mathrm{I}})\right)^{\mathrm{s}}\right) + \boldsymbol{W}^{\oplus_2}_{\oplus_2}(O_{\oplus_2}) + \boldsymbol{q}^*_{\oplus_2/\mathrm{J}_1}\boldsymbol{W}^{\mathrm{J}_1}_{\mathrm{act,J}_1}(O_{\mathrm{J}_1})\boldsymbol{q}_{\oplus_2/\mathrm{J}_1} - \boldsymbol{q}_{\mathrm{J}_2/\oplus_2}\boldsymbol{W}^{\mathrm{J}_2}_{\mathrm{act,J}_2}(O_{\mathrm{J}_2})\boldsymbol{q}^*_{\mathrm{J}_2/\oplus_2},$$
$$(\mathcal{B}_1)_3 = -\boldsymbol{\omega}^{\oplus_3}_{\oplus_3/\mathrm{I}} \times \left(M_{\oplus_3} \star \left((\boldsymbol{\omega}^{\oplus_3}_{\oplus_3/\mathrm{I}})\right)^{\mathrm{s}}\right) + \boldsymbol{W}^{\oplus_3}_{\oplus_3}(O_{\oplus_3}) + \boldsymbol{q}^*_{\oplus_3/\mathrm{J}_2}\boldsymbol{W}^{\mathrm{J}_2}_{\mathrm{act,J}_2}(O_{\mathrm{J}_2})\boldsymbol{q}_{\oplus_3/\mathrm{J}_2} - \boldsymbol{q}_{\mathrm{J}_3/\oplus_3}\boldsymbol{W}^{\mathrm{J}_3}_{\mathrm{act,J}_3}(O_{\mathrm{J}_3})\boldsymbol{q}^*_{\mathrm{J}_3/\oplus_3},$$
$$(\mathcal{B}_1)_4 = -\boldsymbol{\omega}^{\oplus_4}_{\oplus_4/\mathrm{I}} \times \left(M_{\oplus_4} \star \left((\boldsymbol{\omega}^{\oplus_4}_{\oplus_4/\mathrm{I}})\right)^{\mathrm{s}}\right) + \boldsymbol{W}^{\oplus_4}_{\oplus_4}(O_{\oplus_4}) + \boldsymbol{q}^*_{\oplus_4/\mathrm{J}_3}\boldsymbol{W}^{\mathrm{J}_3}_{\mathrm{act,J}_3}(O_{\mathrm{J}_3})\boldsymbol{q}_{\oplus_4/\mathrm{J}_3} - \boldsymbol{q}_{\mathrm{J}_4/\oplus_4}\boldsymbol{W}^{\mathrm{J}_4}_{\mathrm{act,J}_4}(O_{\mathrm{J}_4})\boldsymbol{q}^*_{\mathrm{J}_4/\oplus_4},$$
$$(\mathcal{B}_1)_5 = -\boldsymbol{\omega}^{\oplus_5}_{\oplus_5/\mathrm{I}} \times \left(M_{\oplus_5} \star \left((\boldsymbol{\omega}^{\oplus_5}_{\oplus_5/\mathrm{I}})\right)^{\mathrm{s}}\right) + \boldsymbol{W}^{\oplus_5}_{\oplus_5}(O_{\oplus_5}) + \boldsymbol{q}^*_{\oplus_5/\mathrm{J}_4}\boldsymbol{W}^{\mathrm{J}_4}_{\mathrm{act,J}_4}(O_{\mathrm{J}_4})\boldsymbol{q}_{\oplus_5/\mathrm{J}_4} + \boldsymbol{q}_{\mathrm{G}/\oplus_5}\boldsymbol{W}^{\mathrm{G}}_{\mathrm{ext}}(O_{\mathrm{G}})\boldsymbol{q}^*_{\mathrm{G}/\oplus_5}.$$

Vector $\mathcal{B}_2$ corresponds with r.h.s. of joint constraint eqns,

$$(\mathcal{B}_2)_1 = \Lambda_1 \star \left(\boldsymbol{q}^*_{\mathrm{J}_1/\oplus_1}\boldsymbol{\omega}^{\oplus_1}_{\oplus_1/\mathrm{I}}\boldsymbol{q}_{\mathrm{J}_1/\oplus_1}\right) \times \boldsymbol{\omega}^{\mathrm{J}_1}_{\mathrm{J}_1/\oplus_1},\ (\mathcal{B}_2)_2 = \Lambda_2 \star \left(\boldsymbol{q}^*_{\mathrm{J}_2/\oplus_2}\boldsymbol{\omega}^{\oplus_2}_{\oplus_2/\mathrm{I}}\boldsymbol{q}_{\mathrm{J}_2/\oplus_2}\right) \times \boldsymbol{\omega}^{\mathrm{J}_2}_{\mathrm{J}_2/\oplus_2},$$
$$(\mathcal{B}_2)_3 = \Lambda_3 \star \left(\boldsymbol{q}^*_{\mathrm{J}_3/\oplus_3}\boldsymbol{\omega}^{\oplus_3}_{\oplus_3/\mathrm{I}}\boldsymbol{q}_{\mathrm{J}_3/\oplus_3}\right) \times \boldsymbol{\omega}^{\mathrm{J}_3}_{\mathrm{J}_3/\oplus_3},\ (\mathcal{B}_2)_4 = \Lambda_4 \star \left(\boldsymbol{q}^*_{\mathrm{J}_4/\oplus_4}\boldsymbol{\omega}^{\oplus_4}_{\oplus_4/\mathrm{I}}\boldsymbol{q}_{\mathrm{J}_4/\oplus_4}\right) \times \boldsymbol{\omega}^{\mathrm{J}_4}_{\mathrm{J}_4/\oplus_4}.$$

Tracking Control for End-Effector Pose

- Dual velocity and acceleration of manipulator's end-effector

$$\boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{G/I}} = \boldsymbol{q}^*_{\mathrm{G}/\oplus_1}\boldsymbol{\omega}^{\oplus_1}_{\oplus_1/\mathrm{I}}\boldsymbol{q}_{\mathrm{G}/\oplus_1} + \boldsymbol{q}^*_{\mathrm{G/J}_1}\boldsymbol{\omega}^{\mathrm{J}_1}_{\mathrm{J}_1/\oplus_1}\boldsymbol{q}_{\mathrm{G/J}_1} + \boldsymbol{q}^*_{\mathrm{G/J}_2}\boldsymbol{\omega}^{\mathrm{J}_2}_{\mathrm{J}_2/\oplus_2}\boldsymbol{q}_{\mathrm{G/J}_2} + \boldsymbol{q}^*_{\mathrm{G/J}_3}\boldsymbol{\omega}^{\mathrm{J}_3}_{\mathrm{J}_3/\oplus_3}\boldsymbol{q}_{\mathrm{G/J}_3} + \boldsymbol{q}^*_{\mathrm{G/J}_4}\boldsymbol{\omega}^{\mathrm{J}_4}_{\mathrm{J}_4/\oplus_4}\boldsymbol{q}_{\mathrm{G/J}_4},$$

$$\dot{\boldsymbol{\omega}}^{\mathrm{G}}_{\mathrm{G/I}} = \begin{bmatrix} \boldsymbol{\omega}^{\mathrm{G}}_{\oplus_1/\mathrm{I}} \times \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{G}/\oplus_1} \\ \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{J}_1/\oplus_1} \times \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{G/J}_1} \\ \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{J}_2/\oplus_2} \times \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{G/J}_2} \\ \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{J}_3/\oplus_3} \times \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{G/J}_3} \\ \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{J}_4/\oplus_4} \times \boldsymbol{\omega}^{\mathrm{G}}_{\mathrm{G/J}_4} \\ \mathrm{Q}(\boldsymbol{q}^*_{\mathrm{G/J}_4})_{\mathrm{L}}\dot{Z}_{4,\mathrm{L}}\dot{\Gamma}_4 \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \mathrm{Q}(\boldsymbol{q}^*_{\mathrm{G}/\oplus_1})_{\mathrm{L}}P_{15}^{\mathrm{T}} \\ \mathrm{Q}(\boldsymbol{q}^*_{\mathrm{G/J}_1})_{\mathrm{L}}Z_{1,\mathrm{L}} \\ \mathrm{Q}(\boldsymbol{q}^*_{\mathrm{G/J}_2})_{\mathrm{L}}Z_{2,\mathrm{L}} \\ \mathrm{Q}(\boldsymbol{q}^*_{\mathrm{G/J}_3})_{\mathrm{L}}Z_{3,\mathrm{L}} \\ \mathrm{Q}(\boldsymbol{q}^*_{\mathrm{G/J}_4})_{\mathrm{L}}Z_{4,\mathrm{L}} \end{bmatrix}^{\mathrm{T}} \underbrace{\begin{bmatrix} \dot{\tilde{\omega}}^{\oplus_1}_{\oplus_1/\mathrm{I}} \\ \ddot{\Gamma}_1 \\ \ddot{\Gamma}_2 \\ \ddot{\Gamma}_3 \\ \ddot{\Gamma}_4 \end{bmatrix}}_{=\mu\in\mathbb{R}^{12}},$$

- Globally asymptotically stable pose tracking control

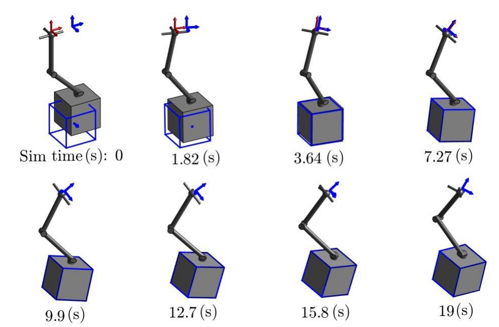


*Figure: End-effector frame (**red**) converges to desired pose frame (**blue**)*

Reference E Trajectory	Reference S Trajectory
$x_{\mathrm{E}}(t) = r_1\cos(\omega_1 t)$	$x_{\mathrm{S}}(t) = r_3\cos(\omega_2 t)$
$y_{\mathrm{E}}(t) = r_1\sin(\omega_1 t)$	$y_{\mathrm{S}}(t) = r_3\sin(\omega_2 t)$
$z_{\mathrm{E}}(t) = r_1\cos(\omega_1 t)$	$z_{\mathrm{S}}(t) = -r_3\cos(\omega_2 t)$
$\phi_{\mathrm{E}}(t) = r_2\cos(\omega_1 t)$	$\phi_{\mathrm{S}}(t) = r_3\cos(\omega_3 t)$
$\theta_{\mathrm{E}}(t) = r_2\sin(\omega_1 t)$	$\theta_{\mathrm{S}}(t) = r_3\sin(\omega_3 t)$
$\psi_{\mathrm{E}}(t) = \sin(\omega_1 t)$	$\psi_{\mathrm{S}}(t) = r_3\sin(\omega_3 t)$
$r_1 = 0.2,\ r_2 = 0.5$ [m] $\omega_1 = -0.2$ [Hz]	$r_3 = -0.25$ [m] $\omega_2 = -0.15,\ \omega_3 = -0.25$ [Hz]

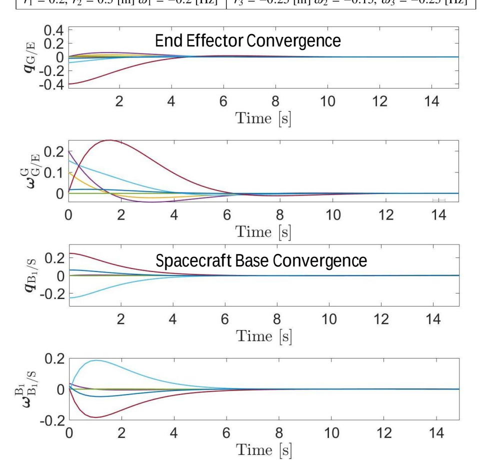


Figure: Error states converge to zero in roughly 10 sec

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