



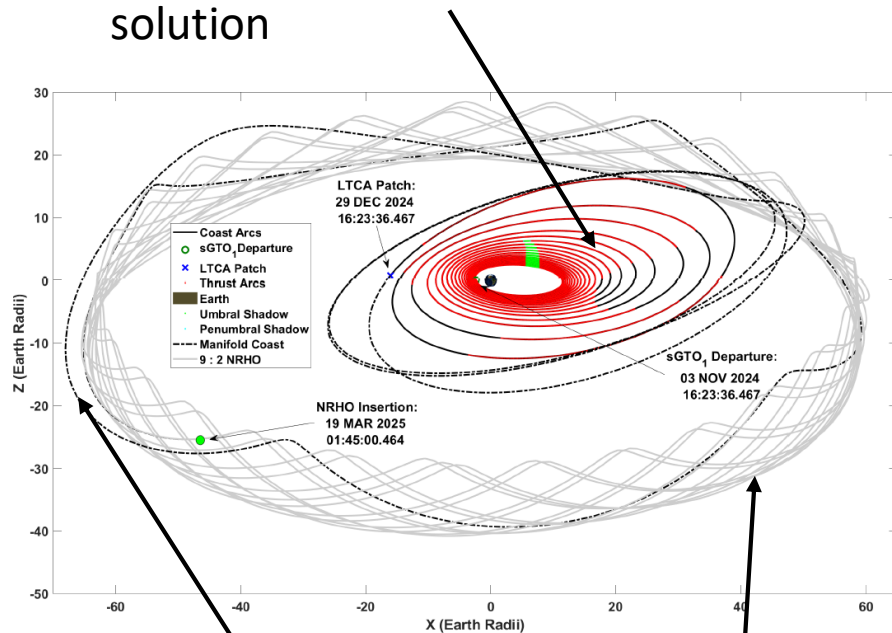


# Tutorial Introduction

## Abstract

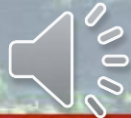
This task aims to develop an approach for designing low-thrust transfers to low-energy lunar orbits with south pole coverage. The approach will use invariant manifolds to improve the quality of the solutions and the speed at which they are produced. Invariant manifolds are groups of ballistic trajectories that travel to and from low-energy orbits. Solving for minimum-fuel low-thrust trajectories can often reveal optimal terminal coast arcs. Therefore, by exploiting the knowledge of known coast solutions to a desired target orbit, we can reduce the necessary optimization while guaranteeing the coast arcs satisfy mission parameters. This effort aims to expand the methods and expertise contained at JPL related to designing low thrust missions in highly perturbed gravitational environments such as, but not limited to, cislunar space. This will put JPL in a beneficial position when missions of this type are necessary in the near future.

Spiral from Earth orbit to manifold perigee using minimum-fuel optimal solution



Long, ballistic coast to target orbit

Target Lunar low-energy orbit



# Problem Description

## a) Context (Why this problem and why now)

- Low-thrust missions attractive for greater science return with less propellant, and are proven (e.g. Dawn)
- Low-thrust trajectory optimization non-trivial, especially in highly perturbed gravitational environments such as cislunar space.
- Such environments enable low energy trajectories
- Can leverage ballistic low-energy trajectories to reduce optimization complexity

## b) Relevance to NASA and JPL (Impact on current or future programs)

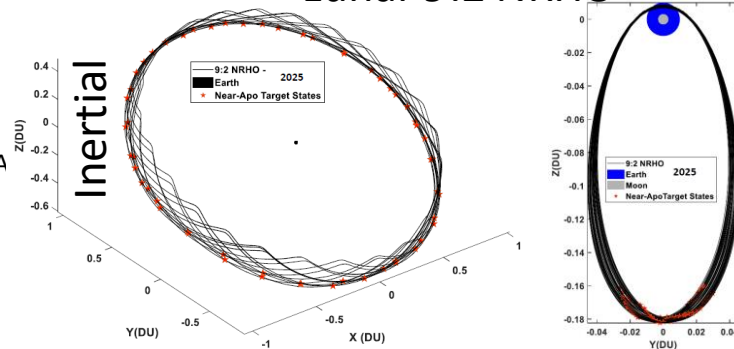
- NASA has plans for human exploration of Lunar south pole
- Supporting missions opportunities likely, such as surface imaging and cargo delivery to Lunar Gateway
- Examining low-thrust transfers through cislunar space to Lunar orbits with south pole coverage is an area that will expand JPL's capabilities of low-thrust mission design and produce opportunities for the Lab in the near future.
- Low energy orbits are becoming more common as target orbits for space exploration, and as such, JPL stands to benefit from having yet another approach to efficient mission design using these orbits.



## Methodology

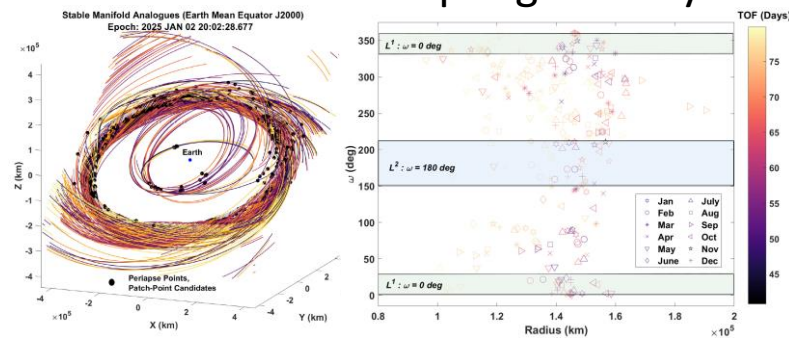
- a) Formulation, theory or experiment description
  - Target orbit: 9:2 Southern Lunar  $L_2$  NRHO (south pole coverage, low energy, NASA Gateway selected)
  - Initial orbit: super-GTO (GTO common for rideshare missions, low-thrust can enable cislunar transfers from GTO)
  - Compute NRHO stable pseudo-manifold in high fidelity model for year 2025
  - Identify promising candidate stable manifold perigee states for each month
    - Based on time of flight, eclipse duration, argument of perigee, perigee radius.
  - Show proposed method works for one candidate NRHO insertion date in 2025
  - Apply proposed method to find minimum fuel solution for each month of 2025
  
- b) Innovation, advancement
  - Replace terminal coast arcs with known manifolds
  - Indirect low-thrust optimization in n-body model, subject to eclipse constraints via homotopy methods to introduce model complexities smoothly

## Quasi-periodic Lunar 9:2 NRHO



Rotating, body-fixed

## Pseudo-manifolds in inertial frame      Pseudo-manifold perigee analysis





## Results

### a) Accomplishments versus goals

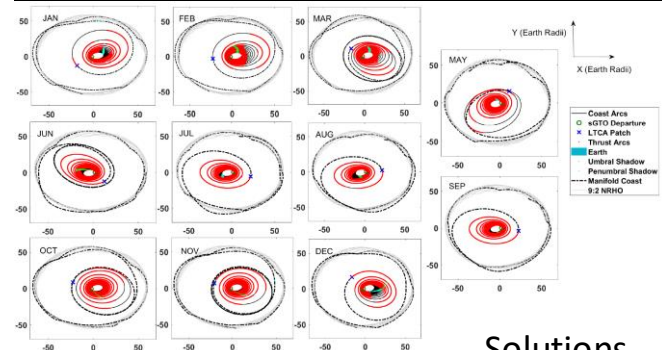
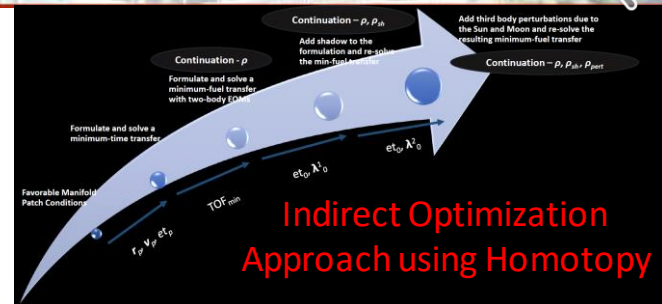
- Computed NRHO stable pseudo-manifold perigee states for year 2025
- Candidate stable manifold perigee states selected for each month in 2025 from large set of solutions
- Applied indirect optimization and composite smoothing control for each step in solution process for sGTO->manifold perigee
- Applied homotopy methods to gradually introduce eclipses and n-body perturbations in dynamical model
- Method proven by finding solutions for each month in 2025

### b) Significance

- Proposed method shown to have valid applications
- Specific application is a current area of interest by NASA
- Alternate approach to trade space exploration in potential other applications using low-thrust, low-energy trajectories
  - Avoid unnecessary optimization in large searches
  - Optimize only part of the trajectory, and guarantee requirements satisfaction by manifold coast arcs

### c) Next steps

- Identify mission concepts that can benefit from this method of optimization



Departure Epoch	Arrival Epoch	$\Delta V$ (m/s)	Max. Eclipse (mins)	#Eclipses
11 OCT 2024 04:17:37.15	25 JAN 2025 18:39:50.11	1641.867	135	19
05 OCT 2024 04:00:27.53	21 FEB 2025 00:19:30.41	1729.059	120	17
03 NOV 2024 16:23:36.47	19 MAR 2025 01:45:00.47	1991.916	126	15
16 JAN 2025 14:12:43.83	04 MAY 2025 00:41:05.43	1956.159	64	8
01 FEB 2025 23:50:15.69	19 JUN 2025 02:58:55.25	2189.803	176	27
02 APR 2025 01:36:17.10	21 JUL 2025 21:10:58.86	1979.373	164	19
01 MAY 2025 06:28:26.86	17 AUG 2025 01:44:34.07	1817.332	124	17
26 MAY 2025 23:20:11.50	12 SEP 2025 01:44:34.07	2054.525	86	17
18 JUN 2025 15:44:17.11	28 OCT 2025 17:18:58.24	2103.743	124	29
11 JUL 2025 04:23:01.44	23 NOV 2025 10:55:04.80	1900.061	130	27
19 AUG 2025 13:18:07.01	19 DEC 2025 14:58:07.03	1772.081	242	28

Solutions for each month in 2025

## Publications and References

### References:

- [1] David E. Lee, "White Paper: Gateway Destination Orbit Model: A Continuous 15 year NRHO Reference Trajectory," JSC-E-DAATN72594, *NTRS - NASA Technical Reports Server*, 2019.
- [2] Rodney L. Anderson, and Martin W. Lo, "Role of invariant manifolds in low-thrust trajectory design," *Journal of guidance, control, and dynamics*, Vol. 32, No. 6, 2009, pp. 1921–1930.
- [3] Sandeep K. Singh, Brian D. Anderson, Ehsan Taheri, and John L. Junkins, "Low-Thrust Transfers to Candidate Near-Rectilinear Halo Orbits facilitated by Invariant Manifolds," *2020 AAS/AIAA Astrodynamics Specialist Virtual Lake Tahoe Conference*, 2020.
- [4] Michael Patterson, John Foster, Thomas Haag, Vincent Rawlin, George Soulas, and Robert Roman, "NEXT: NASA's Evolutionary Xenon Thruster," *38th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit*, 2002, p. 3832.
- [5] Jonathan D. Aziz, "Low-Thrust Many-Revolution Trajectory Optimization," *Ph.D. thesis*, University of Colorado at Boulder, 2018.
- [6] Ehsan Taheri, "Composite Smooth Control Method for Low-Thrust Trajectory Design: Variable Specific Impulse Engine," *AIAA Scitech 2020 Forum*, 2020, p. 2184.
- [7] Ehsan Taheri, John L. Junkins, Ilya Kolmanovsky, and Anouck Girard, "A novel approach for optimal trajectory design with multiple operation modes of propulsion system, part 1," *Acta Astronautica*, 2020.
- [8] Thomas Haberkorn, Pierre Martinon, and Joseph Gergaud, "Low thrust minimum-fuel orbital transfer: a homotopic approach," *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 6, 2004, pp. 1046–1060.
- [9] Hongzhao Liu, and Benson H. Tongue, "Indirect spacecraft trajectory optimization using modified equinoctial elements," *Journal of Guidance, Control, and Dynamics*, Vol. 33, No. 2, 2010, pp. 619–623.
- [10] Ehsan Taheri, Ilya Kolmanovsky, and Ella Atkins, "Enhanced smoothing technique for indirect optimization of minimum-fuel low-thrust trajectories," *Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 11, 2016, pp. 2500–2511.
- [11] John L. Junkins, and Ehsan Taheri, "Exploration of alternative state vector choices for low-thrust trajectory optimization," *Journal of Guidance, Control, and Dynamics*, Vol. 42, No. 1, 2019, pp. 47–64.
- [12] M. Walker, B. Ireland, and J. Owens, "A set modified equinoctial orbit elements," *Celestial mechanics*, Vol. 36, No. 4, 1985, pp. 409–419.
- [13] Binfeng Pan, Ping Lu, Xun Pan, and Yangyang Ma, "Double-homotopy method for solving optimal control problems," *Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 8, 2016, pp. 1706–1720.

### Publications:

- [A] Sandeep Singh, John Junkins, Brian Anderson, and Ehsan Taheri, "Efficient Eclipse-Conscious Low-Thrust Transfers Using Ephemeris-driven Terminal Coast Arcs," to be submitted to *Journal of Guidance, Control, and Dynamics*, American Institute of Aeronautics and Astronautics.