



# Virtual Research Presentation Conference

## Geometric Motion Planners for Highly-Constrained Environments

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**Co-Is:** 347: Saptarshi Bandyopadhyay, Viet Nguyen, & William Seto;  
312: Zaki Hasnain

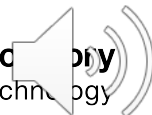
**Program:** Spontaneous Concept

**Acknowledgements:** Ryan Harrod, Michael Trowbridge

Assigned Presentation # RPC-260



**Jet Propulsion Laboratory**  
California Institute of Technology



# Tutorial Introduction

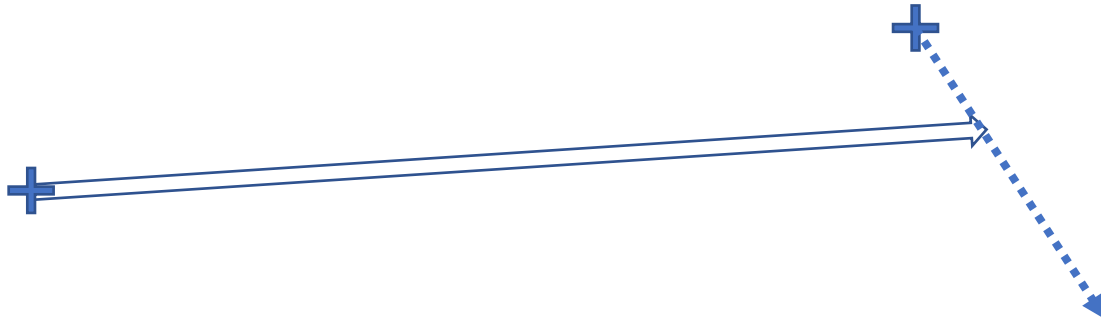
## Abstract

A fundamental problem for **autonomous spacecraft** is **planning motion** without violating **motion constraints** imposed by mission designers or safety concerns. For example: avoiding slewing an instrument to point at the spacecraft body, or not pointing a radiator at a heat source. This is called constrained motion planning.

We evaluate **geometric methods** for generating a **sequence of control setpoints** for a spacecraft or robot in highly constrained environments. If an optimal controller follows these setpoints, the resulting trajectory is guaranteed to be minimal cost.



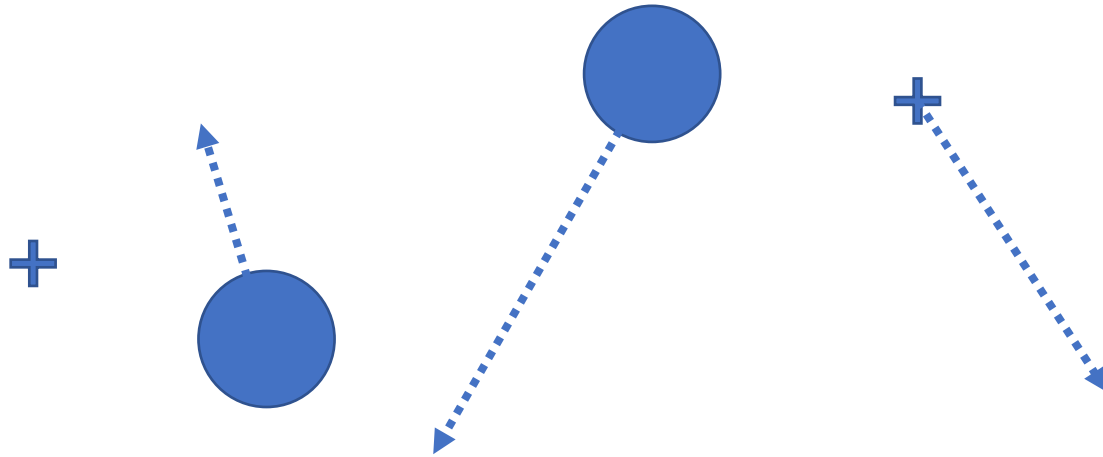
## Example of trajectory optimization



1. Current pose to moving desired pose subject to a maximum change rate!
2. Solution, constraint free, is passed to a control algorithm for path following.



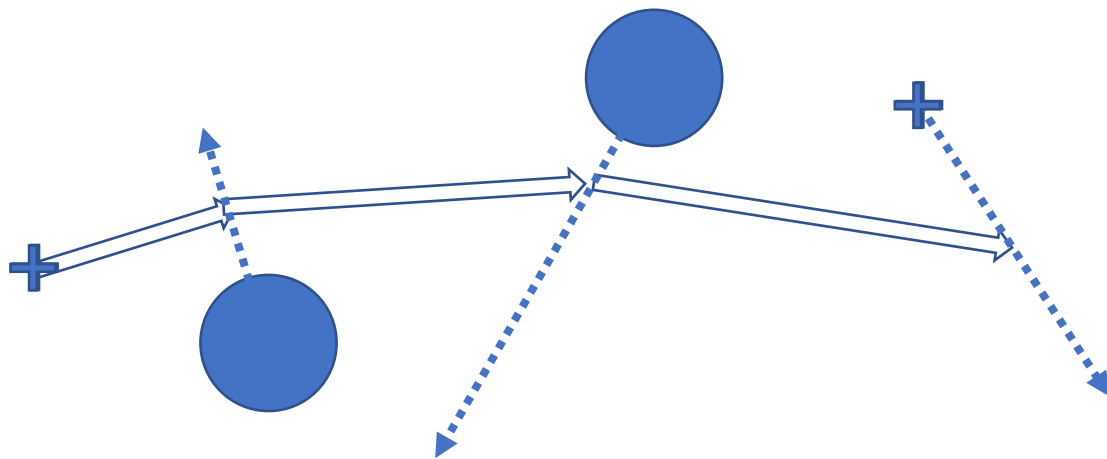
## Example of constrained trajectory optimization



1. Current pose to moving desired pose. Maximum change rate!
2. Solution, constraint free, cannot be followed.



## Example of constrained trajectory optimization



1. Current pose to moving desired pose. Maximum change rate!
2. Solution, with safe segments, is passed to a control algorithm for path following.



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Image by Ryan Harrod



Agility Envelope (actuators)	$f(\vec{\omega}) \leq \gamma$ $f(\dot{\vec{\omega}}) \leq \gamma$
Type I (static hard)	$\mathbf{v}(t)^T \mathbf{w} \leq \cos \theta$
Type II (static soft)	$\int_{t_a}^{t_b}  \mathbf{v}(t)^T \mathbf{w}  dt \leq \phi$
Type III (dynamic hard)	$\mathbf{v}(t)^T \mathbf{w}(t) \leq \cos \theta$
Type IV (mixed)	Formations/thrusters

Table by Michael Trowbridge



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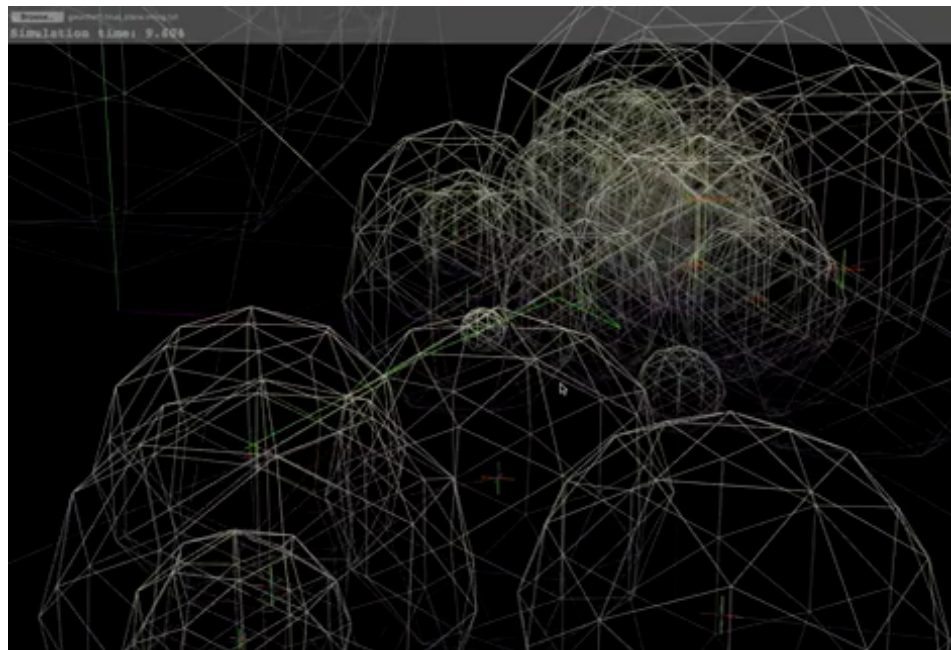


## Methodology

**Formulation:** We treat the problem as planning motion through a controllable space like two or 3d areas, with prohibited control areas that change with time

**Innovation** was to separate paths into geometric components that can be solved in closed form and to stitch them back together to come up with the optimal solution

**We tested the hypothesis** that this method would converge faster and to better solutions by implementing the geometric algorithm and comparing directly on a challenging problem set

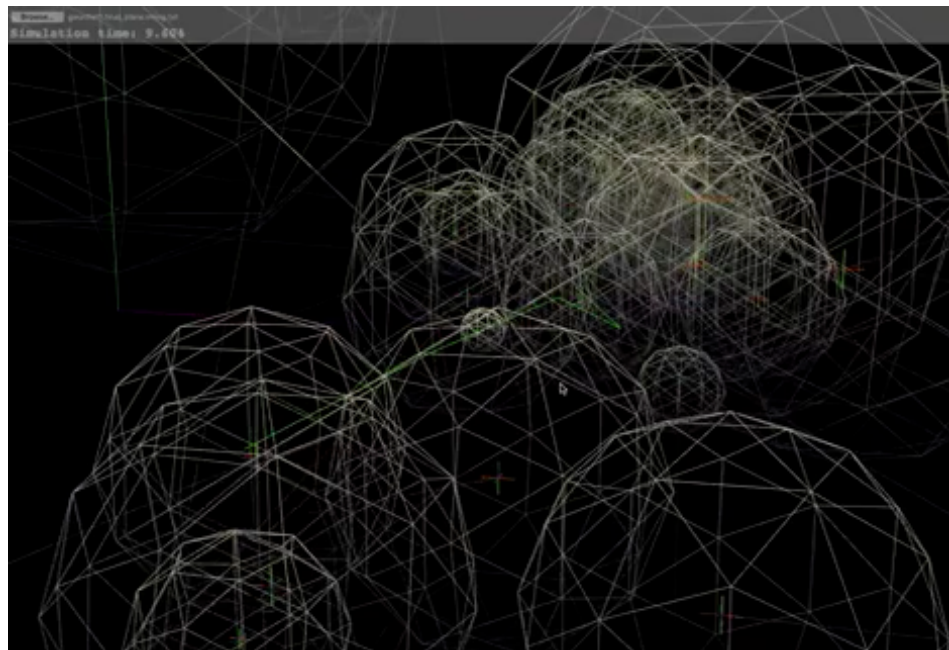


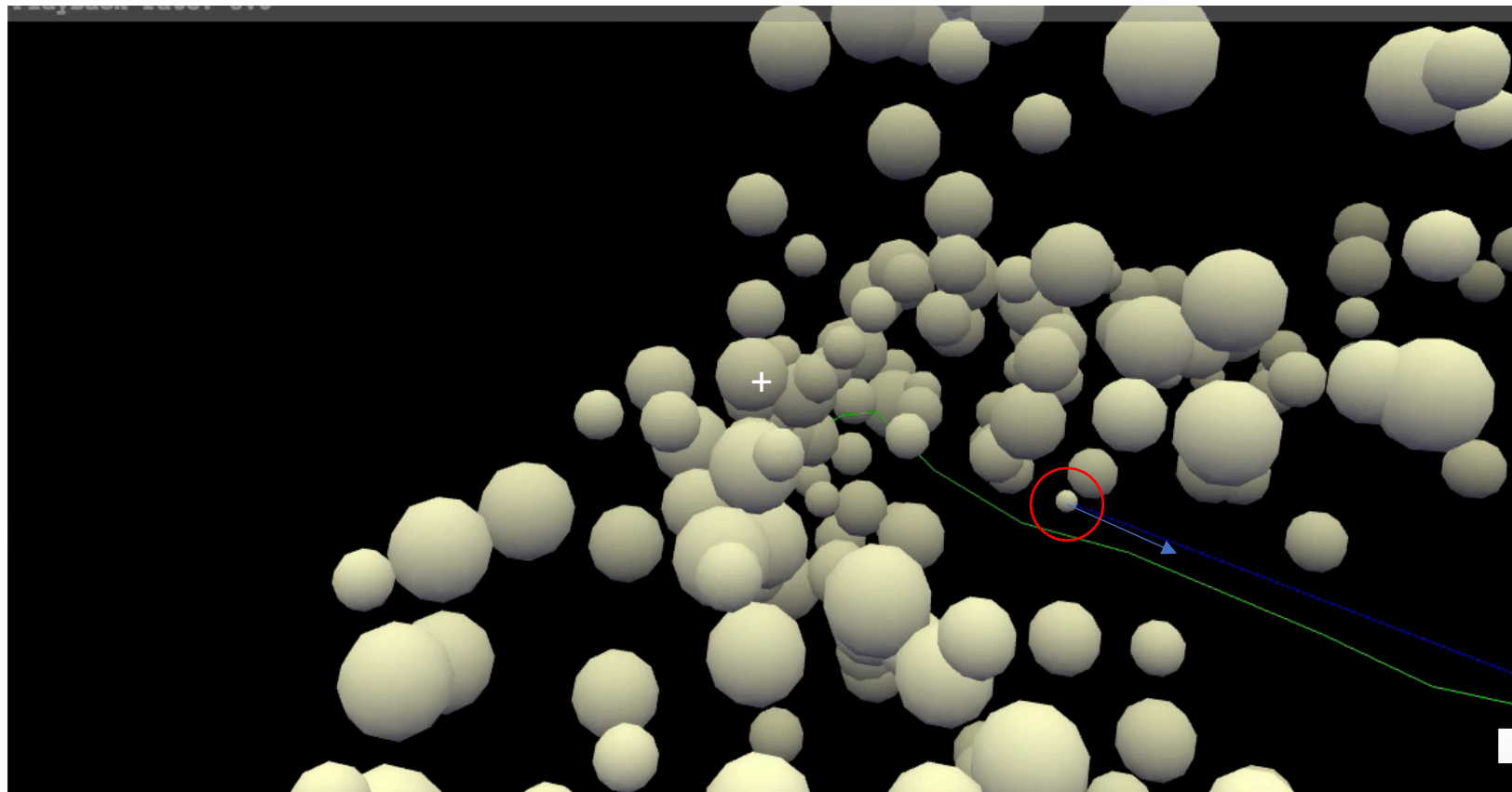
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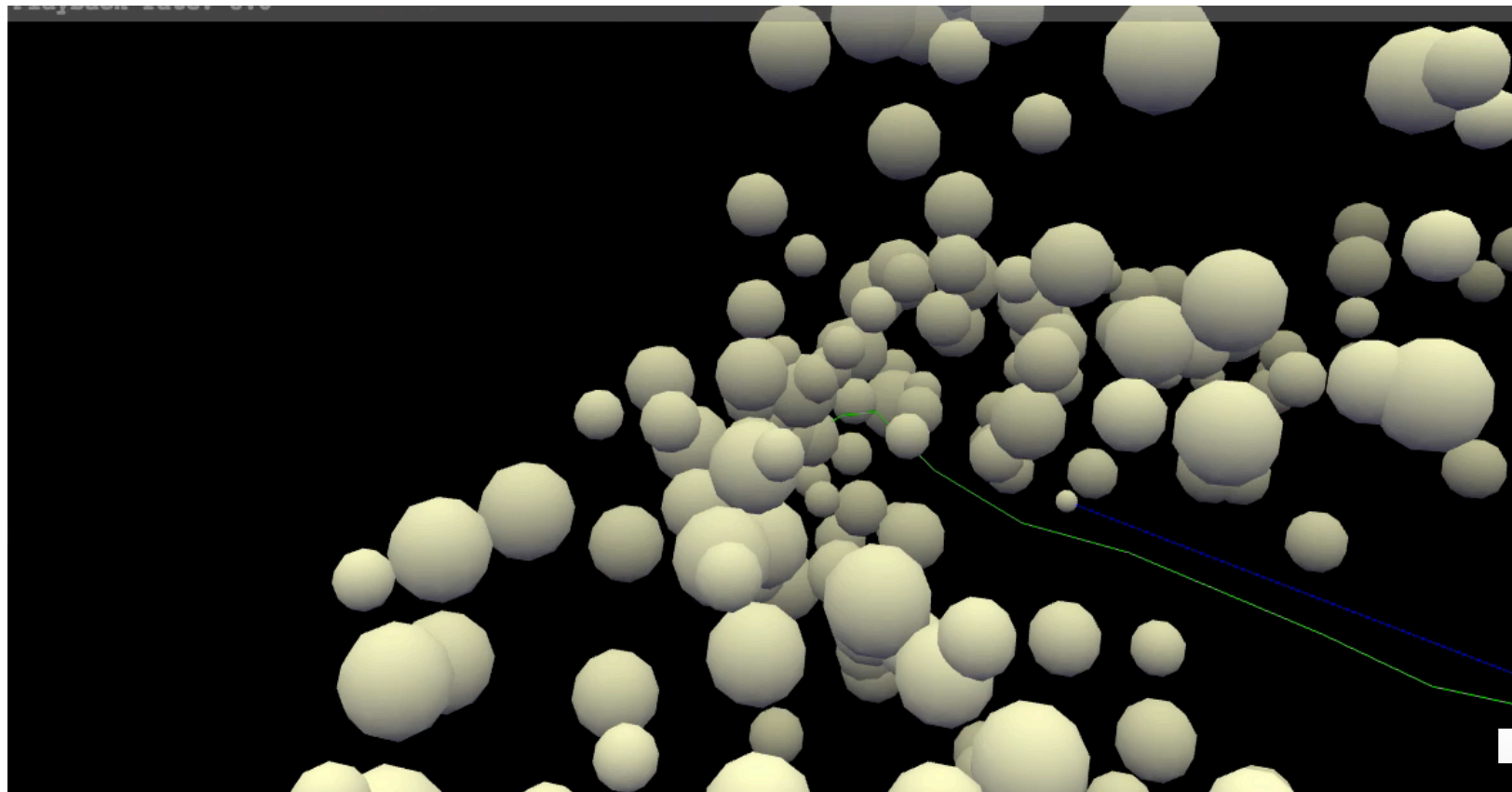
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## Results

- a) Completed implementation of  $R^3$  exact planner
  - a) Easily extended to higher dimensions
- b) Filled missing gap in literature
  - a) Provably optimal solutions
  - b) Provably minimal time cost for fast convergence
- c) ***New guarantees about cost and solution quality!***  
**NTR 51773**

Approach	Type I	Type II	Type III	Type IV	Convergence
Geometric	JPL	☁	JPL	JPL	JPL
Potential Function	✓		✓	☁	☁
Constraint Monitoring	✓	✓	✓	✓	
Randomized	✓	✓	✓	✓	
SDP	✓	✗	✓	✓	✗ ☁

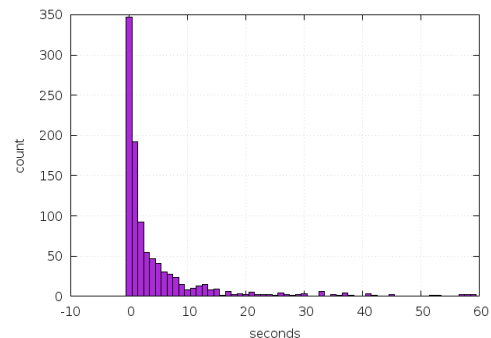
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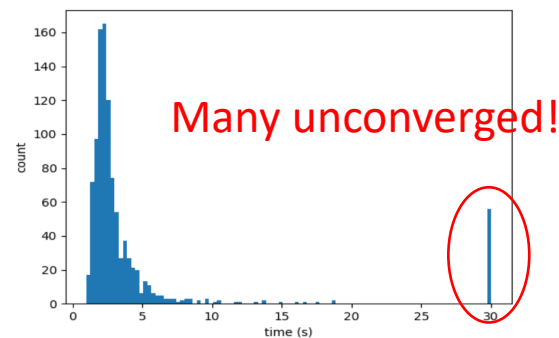
✗ Flaw in proof

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### Ours (median <1)



### OMPL (median 1-2s)



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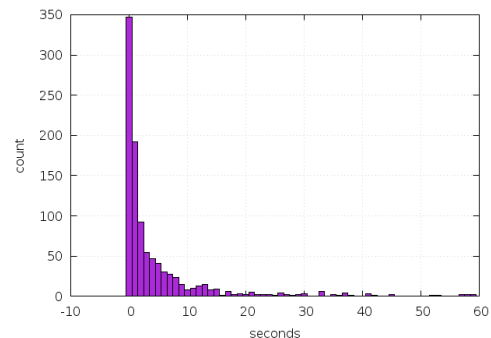
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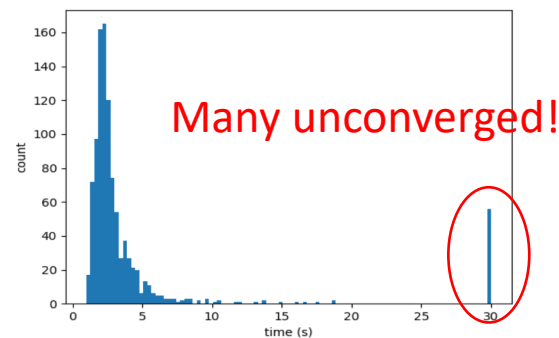
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## Recommended Next Steps

- a) Study in mission context (OCO-3 or a Pre-phase A concept)
- b) Extensions to objective-based planning in addition to constraints
- c) Advanced concepts funding for NSPO / DARPA



# Publications and References

(from AI Group Michael Trowbridge 2020)

Method	Papers	Missions
<b>Direct waypointing (geometric)</b>	Random: Frazzoli et al. 2001, Cheng et al. 2004 A*: Kjellberg and Lightsey, 2013	A*: Bevo-2, ARMADILLO 3U CubeSats
<b>Constraint Monitor</b>	Singh et al. 1997, Raymann et al. 2000	Cassini, Deep-Space 1
<b>Potential Functions/Barrier Functions</b>	McInnes 1994, Spindler 1998, ..., Ramos and Schaub 2018	
<b>Semidefinite Programming</b>	Kim et al. 2004	

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