National Aeronautics and Space Administration

Using an innovative approach with Slepian functions to interpret Cassini Gravity Science data at Saturn.

Principal Investigator: Marzia Parisi (332); Co-Investigators: Dustin Buccino (332)

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Objectives

The objective of this spontaneous R&TD is to perform numerical simulations of the Cassini gravity measurements and demonstrate the applicability of a hybrid mathematical approach that uses both spherical harmonics and Slepian functions for the study of Saturn's gravity field. We demonstrate that the predictions for the high-degree Slepian coefficients lie above the uncertainty level, and can therefore be used to infer the depth of short-scale atmospheric features. These findings open new opportunities for future gravity experiments aboard interplanetary probes, especially in view of missions to Ice Giants, where trajectories are more likely to be highly eccentric and highly inclined (Juno- and Cassini-like), in order to avoid hazardous conditions. The technique provides a way to study the deep winds with the required precision, despite the challenging geometrical constraints. The strong east-west flows are the most recognizable feature of gas and ice giants' atmospheres, and are inextricably connected to their deep interior structure. While the visible surface dynamics are widely understood [1], what happens below the visible clouds is less known and its investigation at greater depths requires measurements of the gravity field. Therefore, the determination of the high-degree gravity field with good accuracy is key for planetary atmosphere studies.

Background

The Cassini spacecraft plunged into Saturn in September 2017, yet a wealth of uninvestigated data will continue to provide scientific breakthroughs for several years. During the Grand Finale, the spacecraft flew between the planet's atmosphere and its rings, including 5 successful gravity passes. The orbit was inclined and highly eccentric, approaching Saturn at low altitudes only for a few hours around pericenter, during which Doppler tracking data of the spacecraft were acquired by ground stations. The gravity field was estimated using the traditional approach of spherical harmonic functions, which has yielded the low-degree, zonal gravity field of Saturn (<J10) with good accuracy, but did not resolve the high-degree, small-scale anomalies [2]. The pericenters occurred over an extremely narrow equatorial belt (Fig. 1), with subsequent loss of orthonormality by spherical harmonics. As a result, the signal from the fine structure of the atmosphere was buried in the noise, with estimated values very close or below the 3σ uncertainty [2].

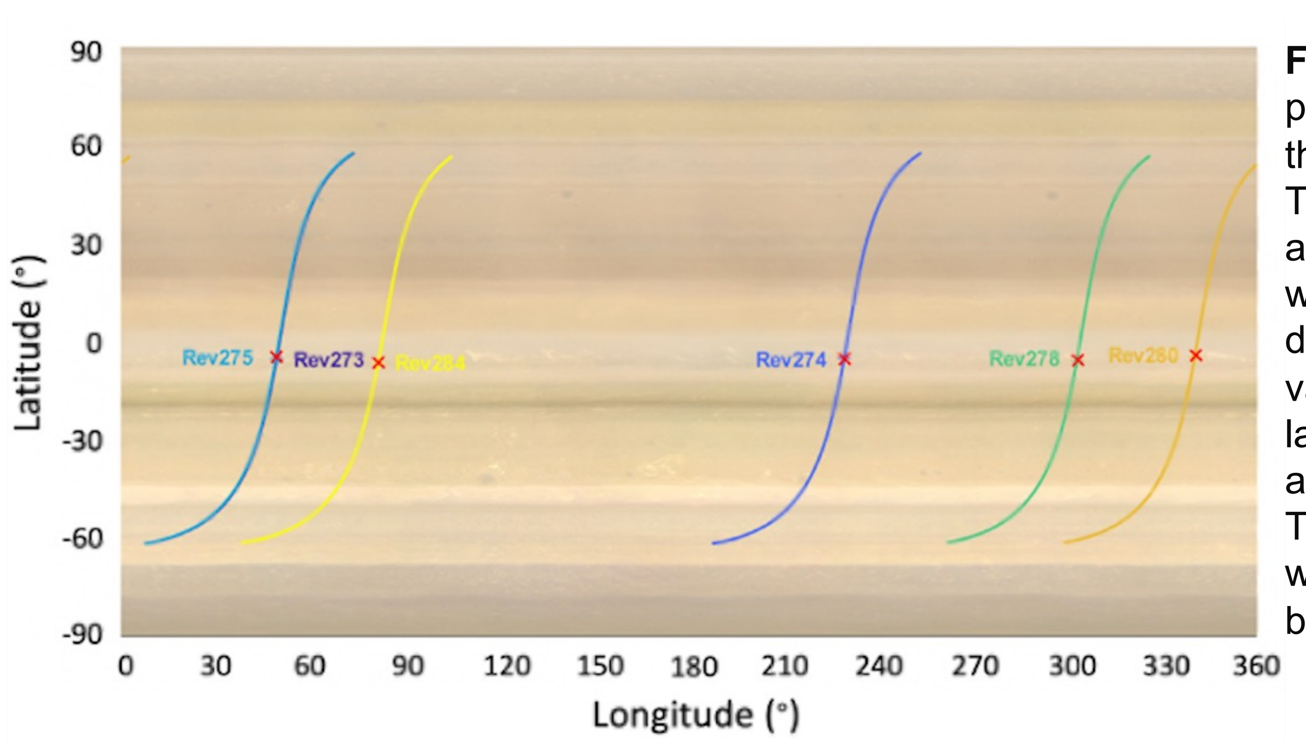

Figure 1. The five gravity passes performed during the Cassini Grand Finale. The distributed closest approaches (red crosses) were sensitive to the low-degree zonal gravitational variations, related to the large-scale, deep atmospheric structure. They were concentrated within a narrow equatorial belt.

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Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

www.nasa.gov

Approach and Results

Slepian functions (Fig. 2) are linear combinations of spherical harmonics that adapt to the actual observational domain, resulting in a more detailed, accurate determination of the local gravity field in the equatorial region [3]. They are defined by solving a maximization problem aimed at augmenting the amount of information carried by the spatially-concentrated data-set of the Cassini observations, carrying little to no energy outside of the selected range of latitudes. Doppler data collected over the five gravity passes of the Cassini Grand Finale were simulated using the most recent spacecraft trajectory reconstruction and information about the set and rising times of the ground stations. The purpose of the simulations is to assess the achievable accuracies in the estimation of the Saturnian gravity coefficients. Once the simulated observables have been generated, it is possible to compare them with the computed Doppler, calculated from the surrounding dynamical environment. The differences are called residuals, which are the basis of the least-squares method for the estimation of physical parameters. The simulations were carried out with the established navigation software MONTE [4].

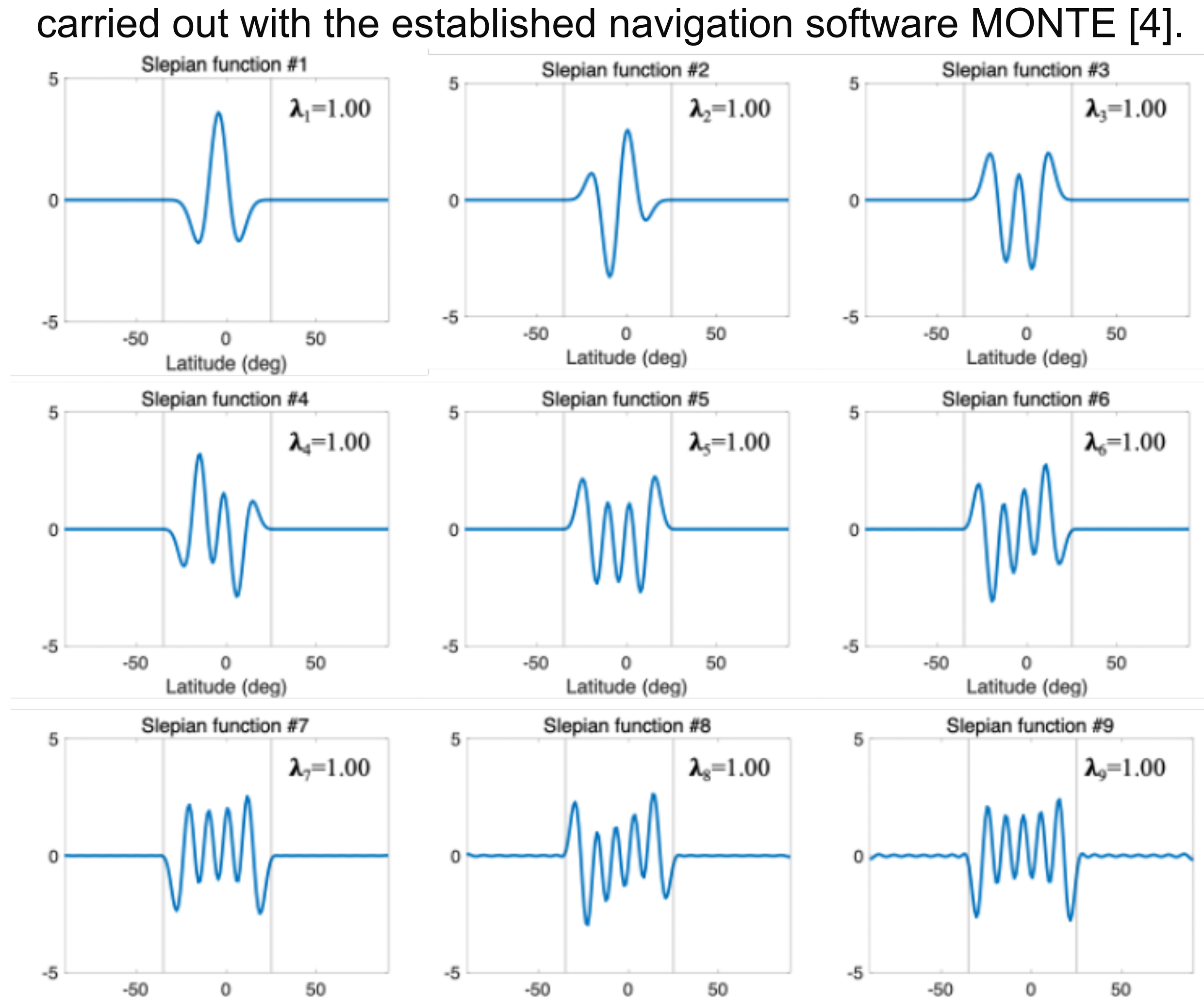

Figure 2. The panels show the 9 normalized Slepian functions that are independent from one another in the selected latitudinal belt (35°S-25°N). They are linear combinations of zonal spherical harmonic functions of degrees N=2,40, and are spatially concentrated within the selected latitudinal band, while they are practically zero (<1% energy) outside [3]. They provide the maximum amount of concentrated information within the chosen domain.

The low-degree gravity field is well-determined up to J_{10} ([2] and Fig. 3), with the exception of J_7 and J_9 that remain close to the uncertainty (3σ) value. The uncertainty curve was generated by carrying out precise numerical simulations of the Cassini Grand Finale gravity measurements. In the hybrid approach (Fig. 4), low-degree spherical harmonics are estimated simultaneously with an optimized number of Slepian functions for the high-degree coefficients, which were created to match the specific domain of the Cassini gravity observations (Fig. 1). Starting from the right side of Fig. 4, the numerical simulations show that by replacing the high-degree harmonic coefficients with optimized Slepian functions, it is possible to significantly decrease the uncertainty (3σ) on the estimation of the small-scale gravity field of Saturn (compared to the right side of Fig. 3). If Slepian coefficients are predicted using thermal wind theory [3], it is found that the central values lie above the uncertainty curve by at least one order of magnitude, if the winds are as deep as several thousand kilometers. The synergy between gravity measurements and thermal wind balance theory provides a physical relationship between the gravity moments and the wind-induced density anomaly, albeit not unique. This work also determined that the Slepian approach allows for the improvement of the estimates of the low-degree gravity field, especially J_7 and J_9, which would further constrain the large-scale, deep circulation of the atmosphere, independently from the choice of the interior model.

PI/Task Mgr Contact
Email: marzia.parisi@jpl.nasa.gov

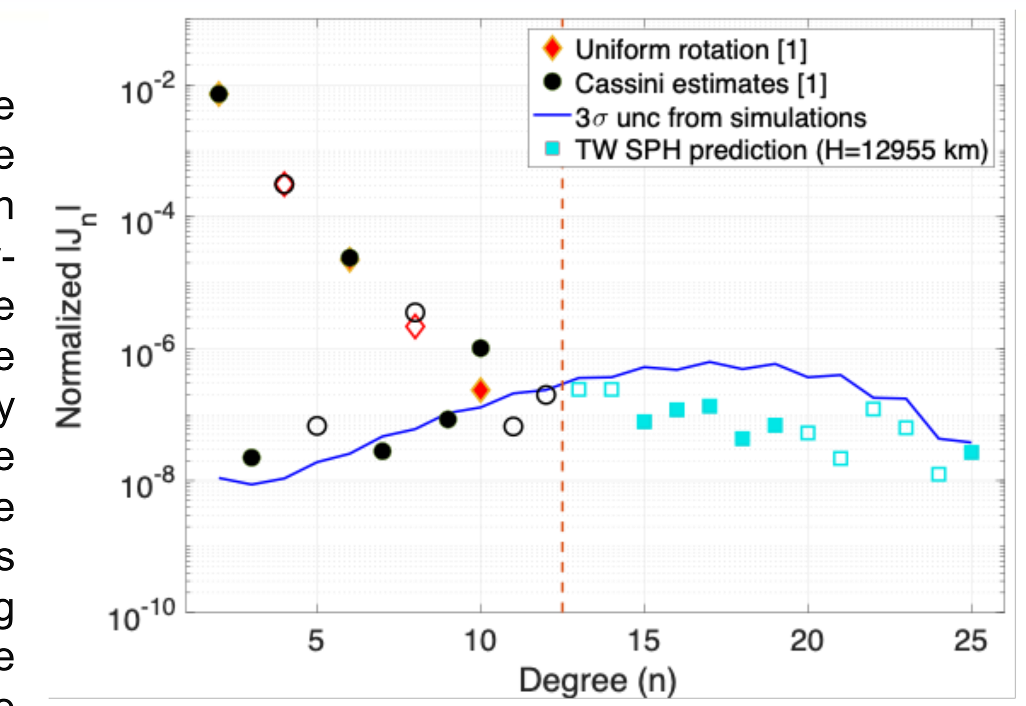

Figure 3. The gravity field of Saturn, in spherical harmonics. The measured coefficients (black circles) are those in [2]. The 3σ formal uncertainties (blue line) and wind gravity predictions (cyan squares) were obtained from current numerical simulations. The red diamonds represent the uniform rotation predictions. Filled and empty markers represent positive and negative coefficients, respectively.

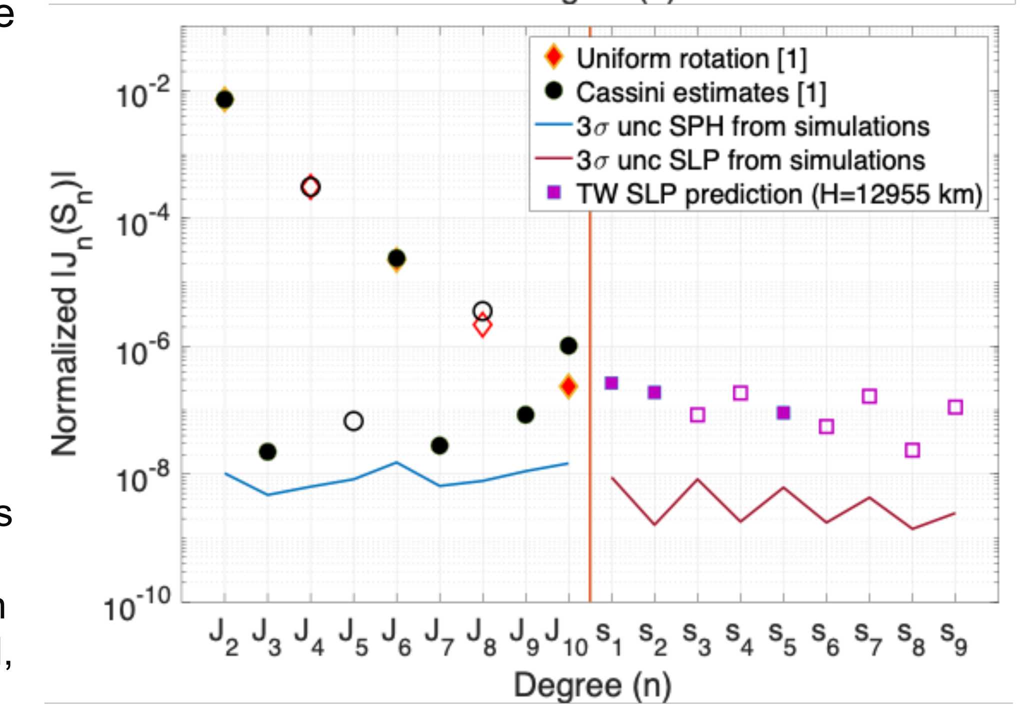

Figure 4. Simulation results using spherical harmonic (left) and Slepian (right) functions. The measured low-degree coefficients (black circles) are those in [2]. The formal uncertainties (blue and burgundy lines) and wind gravity predictions (purple squares) were obtained from original numerical simulations. Filled and empty markers represent positive and negative coefficients, respectively.

Significance/Benefits to JPL and NASA

This study demonstrates the feasibility of determining the local high-degree gravity field of Saturn, by using Cassini Grand Finale observations, which are spatially limited. In addition, it improves significantly (by about one order of magnitude), the accuracy on the estimated values of the low-degree gravity field. Both high-degree and odd zonal harmonics are crucial for the study of the deep atmosphere, as they are dominated by dynamical effects. If followed with an analysis of the actual Cassini radiometric data, the findings will provide both the determination of the depth of atmospheric features at different length scales, as well as new perspectives on the existing models of the interior structure and atmosphere of Saturn. Furthermore, the results will shed light on the sources of internal energy that seem to power the atmosphere of gas and ice giants. The larger number of well determined parameters obtained with the Slepian approach will drastically improve the number of constraints on models of the deep circulations of giant planets. Lastly, the extended knowledge of Saturn's atmosphere and its processes is extremely relevant to the study of exoplanets and other planetary systems.

References

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